Uncertainty on Survival Probabilities and Solvency Capital Requirement: Application to Long-Term Care Insurance

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8 avril 2014 - Version 1.6

Summary

In this paper, we focus on uncertainty issues on disabled lives survival probabilities of LTC insurance policyholders and its consequences on solvency capital requirement. Among the risks affecting long-term care portfolios, special attention is addressed to the table risk, i.e. the risk of unanticipated aggregate mortality, arising from the uncertainty in modeling LTC claimants survival law. The table risk can be thought as the risk of systematic deviations referring not only to a parameter risk but, as well, to any other sources leading to a misinterpretation of the life table resulting for example from an evolution of medical techniques or a change in rules of acceptance. In fine, the idea is to introduce the risk of systematic deviations arising from the uncertainty on the disabled lives death probabilities directly. We analyze the consequences of an error of appreciation on the disabled lives survival probabilities in terms of level of reserves and describe a framework in an Own Risk and Solvency Assessment perspective to measure the gap between the risk profile from the standard formula to the risk analysis specific to the organism.

Keywords. Own Risk and Solvency Assessment, Solvency Capital Requirement, Long-term care insurance, Risk of systematic deviations, Table risk, Semi-parametric model.

Résumé

Dans cet article, on s'intéresse à un modèle permettant de prendre en compte l'incertitude sur la loi de survie d'individus dépendants et les conséquences sur le besoin en fonds propres dans le cadre du dispositif Solvabilité II. Parmi les risques affectant les portefeuilles d'assurance dépendance, une attention particulière est adressée au risque de table, à savoir le risque de mortalité totale imprévue résultant de l'incertitude dans la modélisation de la loi de maintien en dépendance. Le risque de table peut être considéré comme le risque de déviations systématiques. Il intègre le risque d'estimation mais aussi a priori d'autres sources potentielles de méconnaissance de la table résultant par exemple d'une évolution des techniques médicales ou des règles d'acceptation. In fine, l'idée est d'introduire le risque systématique associé à l'aléa sur les probabilités de décès des dépendants directement. Nous analysons les conséquences d'une erreur d'appréciation sur les durées de maintien en termes de niveau de provisions et on décrit un cadre utilisable dans une logique ORSA pour mesurer l'écart entre le profil de risque issu de la formule standard et celui issu d'une analyse des risques spécifiques à l'entité.

Mots-clés. ORSA, SCR, Assurance dépendance, Risque de déviations systématiques, Risque de table, Modèle semi-paramétrique.

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1 Introduction

The important change in Solvency II is the general reform of assessment of the insurer solvency. The solvency is not only a matter of equity, but a meaningful process of accountability in which the insurer must monitor its activities through an adapted system of management and risk control. The ORSA (Own Risk and Solvency Assessment) carries the key issues of the reform of Solvency II. It is the set of processes that contribute to the regular assessment of the overall internal solvency of the company as an integral part of the business strategy and taking into account the specific risk profile of the insurer. This identification of the specific risk profile of the company is the cornerstone of an effective governance. The ORSA allows to shift from a logic of retrospective risks control to a logic of steering by monitoring the risks which incorporates the solvency. By identifying factors that may affect the current and future solvency, the ORSA offers the opportunity to the insurer to respond promptly and effectively to the economic context.

The need for a sound assessment of a LTC (Long-Term Care) insurer's risk profile suggests a comprehensive approach to the liabilities of this particular life insurance business. LTC insurance is a range of financial services allowing the person to cope with his or her loss of mobility and autonomy in his or her activity of daily living. The French insurance market for LTC is the second largest worldwide market with around 3 millions policyholders and a growth rate of 15% per year, see Kessler (2008) for a study on the french LTC insurance market. The public coverage of LTC is derived not only from a long french tradition of intervention concerning social assistance, but also from the great diversity of actors and sources of financing, see Courbage and Roudaut (2011). In addition private insurance has developed.

Most of the actuarial publications on this topic focus on the construction of models of projected benefits, see Gauzère *et al.* (1999) and Deléglise *et al.* (2009) and the assessment of transition probabilities to model the life-history of LTC patients, see Czado and Rudolph (2002) and Helms *et al.* (2005).

In contrast, in our study, we have no exogenous information about the LTC insured population, in terms of gender, place, or level of care. We observe only the aggregated exposition and number of deaths over two dimensions. These are the age of occurrence of the disability and the duration of the disability. Hence, LTC claimants belong only to one state of severeness (heavy claimants). The pricing and reserving as well as the management of LTC portfolios are very sensitive to the choice of the mortality table adopted. In addition, the construction of such table is a difficult exercise, see Tomas and Planchet (2013).

In this article, we analyze the consequences of uncertainty on the disabled lives survival probabilities of LTC insurance policyholders in terms of level of reserves. We describe a framework, in an ORSA perspective, to measure the gap between the risk profile from the standard formula to a risk analysis specific to the insurer. The mortality law as well as the disability law are assumed to be known. The model could also be applied to the disabilities but it is not the subject to this article. It would lead to a more complex notation and would show no major complications in assessing the basic equation of the variance of the insurer's liabilities.

The article is organized as follow. Section 2 has still an introductory purpose and makes precise the notation used in the following. In Section 3, the risk of random fluctuations is briefly sketched. The table risk and an application to the computation of the SCR (Solvency Capital Requirement) are then addressed. Section 4 presents the numerical application. Finally, some remarks in Section 5 conclude the paper.

2 Notation and reserves valuation

2.1 Notation

We consider a LTC insured population of n individuals. For a policyholder aged x at the time of computation, we note Υ_x the time before the person losses his or her mobility and autonomy, i.e. the time of occurrence of the disability. In other words, Υ_x is the random variable such that the disability appears at age $x + \Upsilon_x$. The law of Υ_x is known from the mortality law of the policyholders and the disability law. More precisely, the probabilities, ρ_{x+t}, defining the law of Υ_x are

$$\rho_{x+t} = \mathbb{P}\big[\Upsilon_x = t\big] = \varrho_x \prod_{v=0}^{t-1} \big(1 - q_{x+v}^{\text{ins}}\big)\big(1 - \varrho_{x+v}\big),$$

where ϱ_x denotes the probabilities defining the disability law at age x and q_x^{ins} the probabilities of death derived from the regulatory market table.

We assume only one state of severeness, the heavy claimant, and recovery is excluded. The mortality of LTC claimants is analyzed over both age of occurrence of the disability $x + \Upsilon_x$, which is an explanatory variable, and duration of the disability (or seniority) denoted by u, which is the duration variable.

We represent by $T_u\big(x + \Upsilon_x\big)$ the remaining disabled lifetime of a policyholder when the disability occurred at age $x + \Upsilon_x$ with the duration of the disability u, while the survival disabled lives probability is :

$$\mathbb{P}\big[T_u\big(x + \Upsilon_x\big) > t\big] = \mathbb{P}\big[T\big(x + \Upsilon_x\big) > u + t \big| T\big(x + \Upsilon_x\big) > u\big] = {}_tp_u^{\text{claim}}\big(x + \Upsilon_x\big).$$

Hence, a policyholder with the disability occurring at age $x + \Upsilon_x$ and duration of the disability u will decease at the duration $u + T_u\big(x + \Upsilon_x\big)$.

The distribution function of $T_u(x+\Upsilon_x)$ is

$$ {}_tq_u^{\text{claim}}(x+\Upsilon_x) = 1 - {}_tp_u^{\text{claim}}(x+\Upsilon_x) = \mathbb{P}\left[T_u(x+\Upsilon_x) \leq t\right] = \mathbb{P}\left[T(x+\Upsilon_x) \leq u+t | T(x+\Upsilon_x) > u\right]. $$

2.2 Disabled reserves valuation

We note $\Psi(x, \Upsilon_x)$ the random amount of the insurer's liabilities for a policyholder aged x at the time of the computation. It is the sum of the future cash flows for an policyholder aged x discounted with rate r,

$$ \Psi(x, \Upsilon_x) = \sum_{t=1}^{+\infty} (1+r)^{-(t+\Upsilon_x)}\, c\, \mathbb{I}_{]t;+\infty[}\big(T_u(x+\Upsilon_x)\big), $$

where c represents the individual care delivered during the period t.

Two uncertainties apply to the law of Ψ_x. On one hand we have the uncertainty related to the time of occurrence of the disability, on the other hand, there is the uncertainty associated with the duration of the disability. In this article, we only focus on the risk of systematic deviations referring to a misinterpretation of the disabled lives survival probabilities. In consequence we can integrate with respect to the law of Υ_x to obtain,

$$ \begin{aligned} \Psi_x = \mathbb{E}_{\Upsilon_x}\left[\Psi(x, \Upsilon_x)\right] &= c \sum_{\upsilon \geq 1} \rho_{x+\upsilon} \sum_{t=1}^{+\infty} (1+r)^{-(t+\upsilon)}\, \mathbb{I}_{]t;+\infty[}\big(T_u(x+\upsilon)\big) \\ &= c\, \psi \end{aligned} $$

It leads for a portfolio of n insured individuals to

$$ \Lambda = \sum_{i=1}^{n} c_i\, \psi_i. $$

In the following, we are interested in the expectation of Λ, i.e. the disabled reserve, and more generally to the law of Λ. The reserve has a simple form :

$$ \begin{aligned} \mathbb{E}[\Lambda] &= \sum_{i=1}^{n} c_i \sum_{\upsilon \geq 1} \rho_{x+\upsilon} \sum_{t=1}^{+\infty} (1+r)^{-t}\, \mathbb{P}\left[T_u(x+\upsilon) > t\right]. \\ &= \sum_{i=1}^{n} c_i\, \mathbb{E}[\psi] \end{aligned} $$

When provisioning the amount $\mathbb{E}[\Lambda]$, the insurer faces adverse deviations due to two distinct factors :

i. The random fluctuations of the observed disabled lives mortality rates around the relevant expected values, i.e., the fitted mortality rates, which are consequences of the finite size of the population exposed to the risk. The risk of random fluctuations (often called process risk) is diversifiable (one should better say pooled). Its financial impact decreases, in relative terms, as the portfolio size increases.

ii. The inaccuracy of the underlying disabled lives survival law, from which the probability $\mathbb{P}[T_u(x+\upsilon) > t]$ are derived, is called the table risk. This is the risk of unanticipated aggregate mortality, arising from the uncertainty in modeling the LTC claimants' survival law. The table risk can be thought of as the risk of systematic deviations referring not only to a parameter risk but, as well, to any other sources leading to a misinterpretation of the life table resulting for example from an evolution of medical techniques or a change in rules of acceptance. The risk of systematic deviations cannot be hedged by increasing the portfolio size. Actually, in relative terms, its severity does not reduce as the portfolio size increases, since deviations concern all the insureds in the same direction.

In the following, the risk of random fluctuations is briefly sketched. The table risk and its impact on the SCR are then addressed.

3 The risk components and SCR

3.1 Risk of random fluctuations

To incorporate the uncertainty arising from the random fluctuations of the observed disabled lives mortality rates around the relevant expected values, i.e. the fitted mortality rates, we can construct a confidence interval of the disabled reserve around its expected value.
Due to the assumption of independence between policyholders and as we can reasonably assume that the individual cash flows ψ_i are bounded by a constant, i.e. that the set $\{\psi_i\}_{i=1}^n$ is uniformly bounded, the limit distribution of Λ is gaussian :

$$\frac{\Lambda - \mathbb{E}\big[\Lambda\big]}{\sigma_\Lambda} \xrightarrow[n\to+\infty]{} \mathcal{N}(0,1),$$

where σ_Λ denoting the standard deviation of Λ. Hence, we can approximate the distribution of the discounted cash flows and we can easily derive quantiles and confidence interval. For example, Λ falls in the random interval with approximately $(1-\alpha)$ coverage probability,

$$\mathcal{I}_\Lambda = \mathbb{E}\big[\Lambda\big] \pm \Phi^{-1}_{1-\alpha/2} \times \sigma_\Lambda,$$

with $\Phi^{-1}_{1-\alpha/2}$ chosen as the $(1-\alpha/2)$ quantile of the standard normal distribution.

To assess the two first moments of Λ, since

$$\mathbb{E}\big[\Lambda\big] = \sum_{i=1}^{n} c_i\,\mathbb{E}\big[\psi\big] \quad \text{and} \quad \mathbb{V}\big[\Lambda\big] = \sum_{i=1}^{n} c_i^2\,\mathbb{V}\big[\psi\big], \tag{1}$$

we only need to know the expectation and variance of

$$\psi = \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \sum_{t=1}^{+\infty} (1+r)^{-(t+\upsilon)}\,\mathbb{I}_{]t;+\infty[}\big(T_u\big(x+\upsilon\big)\big)$$

One easily finds :

$$\mathbb{E}\big[\psi^2\big] = \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \left(\sum_{t=1}^{+\infty} \frac{S_{x+\upsilon}(t)}{(1+r)^{2(t+\upsilon)}} + 2 \times \sum_{t=2}^{+\infty} \frac{1}{r} \left(\frac{1}{(1+r)^{t+\upsilon}} - \frac{1}{(1+r)^{2(t+\upsilon)+1}} \right) S_{x+\upsilon}(t) \right),$$

where $S_{x+\upsilon}(t)$ denotes the survival function when the disability occurred at age $x+\upsilon$ for duration t.

However, it is simpler to start with a continuous expression of ψ,

$$\psi = \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \int_0^{+\infty} \exp(-\tau\, t)\,\mathbb{I}_{]t;+\infty[}\, dt \quad \text{with} \quad \tau = \ln(1+r).$$

It leads to

$$\mathbb{E}[\psi] = \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \int_0^{+\infty} \exp(-\tau\, t)\; S_{x+\upsilon}(t)\; dt \approx \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \sum_{t\geq 1} \exp(-\tau\, t)\; S_{x+\upsilon}(t), \tag{2}$$

$$\text{and} \quad \mathbb{E}[\psi^2] = \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \frac{2}{\tau} \int_0^{+\infty} \big(1-\exp(-\tau\, t)\big) \exp(-\tau\, t)\; S_{x+\upsilon}(t)\; dt$$

$$\approx \sum_{\upsilon\geq 1} \rho_{x+\upsilon} \frac{2}{\tau} \sum_{t\geq 1} \big(1-\exp(-\tau\, t)\big) \exp(-\tau\, t)\; S_{x+\upsilon}(t). \tag{3}$$

Note that the discrete approximation of the expectation is identical to the one obtained previously, while the approximation of the variance differs slightly.

We will not go further in assessing the risk of random fluctuations. Extensive studies have discussed the issue, see among others Milevsky and Promislow (2001), Dahl (2004), Biffis (2005), and Ballotta and Haberman (2006) for the valuation of random fluctuations. In the following we focus on the table risk arising from the uncertainty in modeling the disabled lives survival law.

3.2 Table risk

The choice of the mortality table adopted has a crucial impact on the pricing and reserving as well as the management of LTC portfolios. Tomas and Planchet (2013) have shown that the construction of such table is a difficult exercise for the following reasons :

i. The mortality law of LTC claimants consists of a mixture of pathologies, and non-monotonic phenomena appear.

ii. LTC portfolios are relatively small, and the estimation of crude death rates is very volatile.

iii. Due to the strong link between the age of occurrence of the disability and the related disability, it is usual to construct a mortality surface.

iv. Disabled lives mortality rates decrease very rapidly during the first months of the duration of the disability. In consequence, the first year is often difficult to integrate with the usual parametric approaches.

Unlike the risk of random fluctuations, the table risk is systematic, due to the fact that it concerns aggregate mortality. It is realized when deviations from expected mortality are observed along the duration of the care. Here, the idea is to introduce the risk of systematic deviations arising from the uncertainty on the disabled lives death probability directly with a semi-analytic approach.

One of the simpler ways to introduce an uncertainty on the expected mortality is to add a disturbance on the logits of the fitted disabled lives death probabilities, see Planchet and Thérond (2011).

We note by $\widehat{q}_u(x+\upsilon)$ and $\widetilde{q}_u(x+\upsilon)$ the fitted and disturbed disabled lives death probability at duration u for the age of occurrence $x+\upsilon$ respectively. Then,

$$\operatorname{logit} \widetilde{q}_u(x+\upsilon) = \ln\left(\frac{\widetilde{q}_u(x+\upsilon)}{1-\widetilde{q}_u(u+\upsilon)}\right) = \ln\left(\frac{\widehat{q}_u(x+\upsilon)}{1-\widehat{q}_u(x+\upsilon)}\right) + \epsilon, \tag{4}$$

where ϵ is a variable centered which we suppose to be gaussian in the following.

Equivalently,

$$\widetilde{q}_u(x+\upsilon) = \frac{a \times \exp\big(\text{logit } \widehat{q}_u(x+\upsilon)\big)}{1 + a \times \exp\big(\text{logit } \widehat{q}_u(x+\upsilon)\big)}, \quad \text{with} \quad \epsilon = \ln a. \tag{5}$$

The disturbance is controlled by the volatility of ϵ, noted σ_ϵ. We will vary σ_ϵ from 1 to 20 %, and measure the uncertainty on the expectation of the remaining disabled lifetime of a policyholder in computing the relative difference, denoted by δ, between the expectation and, $\phi_{95\%}$, the 95 % quantile of the remaining disabled lifetime. If we consider that the care costs 1 and a zero discount rate, it leads to

$$\delta = \frac{\phi_{95\%}\left(\mathbb{E}\big[T(x+\upsilon)\,|\,a\big]\right) - \mathbb{E}\left[\mathbb{E}\big[T(x+\upsilon)\,|\,a\big]\right]}{\mathbb{E}\left[\mathbb{E}\big[T(x+\upsilon)\,|\,a\big]\right]} = \frac{\phi_{95\%}\left(\mathbb{E}\big[T(x+\upsilon)\,|\,a\big]\right) - \mathbb{E}\big[T(x+\upsilon)\big]}{\mathbb{E}\big[T(x+\upsilon)\big]}. \tag{6}$$

In fine, the risk of systematic deviations arising from the uncertainty is introduced on the disabled lives death probability directly.

3.3 Application to the SCR

The current standard requirements for the Solvency II life risk module have been specified in QIS5, CEIOPS (2010, pp.147-163). QIS5 prescribes a SCR which accounts explicitly for the uncertainty arising from the systematic deviations and parameters estimation but not for the random fluctuations and process risk (severity of claims). In fact, the process risk has been disregarded as not significant enough, and has been included in the systematic and parameter risk component, in order to simplify the standard formula.

The severity of the risk of random fluctuation decreases, in relative terms, as the portfolio size increases. Hence, we can suppose, for sufficient exposure, that the risk of systematic deviations and parameter risk have a larger financial impact. Here, our aim is then to measure the relevancy of the shocks described in the QIS5 specification with the specific risks supported by the insurer in an ORSA perspective. Under Solvency II, LTC is considered the same way as life business. In the following, we are only interested about the longevity shocks (a decrease of 20 % in mortality rates for each age) applied to the LTC claimants only, not to the overall insured population mortality.

Computing the quantiles of the distribution of the sum of the discounted cash flows through expression (6) gives a biased evaluation of the SCR, because it does not take in account the limitation of the projection (computed to infinity), and the risk margin. The literature on SCR approximation is very extensive, see among others Bauer *et al.* (2010) and Stevens *et al.* (2010). However, here we use the general approximation of Guibert *et al.* (2010) in order to have a simple framework to compute the SCR in the presence of a systematic risk factor affecting the poolable risk.

Following Guibert *et al.* (2010, Section 3), we use the general approximation

$$\text{SCR}_1 = \frac{\dfrac{\text{VaR}_{99.5\,\%}(\chi)}{\text{BEL}_0} - 1}{1 - \alpha\, D_0 \left(\dfrac{\text{VaR}_{99.5\,\%}(\chi)}{\text{BEL}_0} - 1\right)} \times \text{BEL}_0 \quad \text{with} \quad \chi = \frac{F_1 + \text{BEL}_1}{1 + R_1},$$

where F_1 denotes the cash flows payable at month 1, R_1 is the return on assets at month 1 and D_0 the duration of the liability (one month). The best estimate of the reserve at month 0 and 1 are denoted by BEL_0 and BEL_1 respectively.

The variable χ can be interpreted as the economic liability according to the assets allocation, see Planchet and Thérond (2007), while

$$\frac{\mathrm{VaR}_{99.5\,\%}(\chi)}{\mathrm{BEL}_0} - 1$$

represents the one-month solvency ratio without taking in account the duration of capital commitment.

The law of the variable χ can be reasonably approximated by the sum of the discounted cash flows, i.e. Λ, which is, conditionally to the disturbance, (approximatively) gaussian, see Guibert *et al.* (2010, Section 3.2) :

$$F_\Lambda(x) = \mathbb{P}\left[\Lambda \leq x\right] = \mathbb{E}\left[\mathbb{P}\left[\Lambda \leq x | a\right]\right] \xrightarrow[n\to+\infty]{} \int \Phi\left(\frac{x-\mu(a)}{\sigma(a)}\right) F_a(da).$$

In practice, we approximate this function by Monte Carlo simulations on the basis of a sample of the variable a :

$$F_\Lambda(x) \approx F_K(x) = \frac{1}{K}\sum_{k=1}^{K} \Phi\left(\frac{x-\mu(a_k)}{\sigma(a_k)}\right),$$

where K denoted the number of simulations. Then, a quantile ρ is derived by solving the equation $F_K(x_\rho) = \rho$ numerically. This model have been used to integrate risk of a pandemia into an internal model in the Solvency II framework by Planchet (2013).

The moments of Λ are derived in expression (1). If we consider that the monthly care costs 1 and a zero discount rate, it leads, for a portfolio of n LTC claimants to

$$\mu_a = \mathbb{E}\left[\Lambda | a\right] = n \times \mathbb{E}\left[\Psi | a\right] \quad \text{and} \quad \sigma_a = \sigma_{\Lambda\,|\,a} = \sqrt{n \times \mathbb{V}\left[\Psi | a\right]},$$

$$\text{with} \quad \mathbb{E}\left[\Psi | a\right] \approx \sum_{v\geq 1} \rho_{x+v} \sum_{t\geq 1} S_{x+v}(t\,|\,a) \quad \text{and} \quad \mathbb{V}\left[\Psi | a\right] \approx 2\sum_{v\geq 1} \rho_{x+v} \sum_{t\geq 1} t\, S_{x+v}(t\,|\,a) - \left(S_{x+v}(t\,|\,a)\right)^2,$$

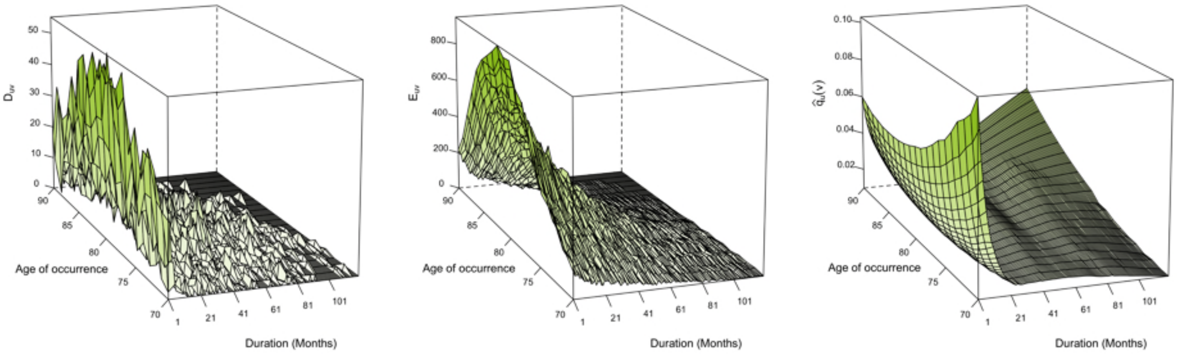
following expressions (2) and (3).

Having an discounted rate $r > 0$ will simply give more weight to the nearest cash flows and therefore will tend to lower volatility of the estimators.

We can then compute the ratio between the SCR and the best estimate of the reserve as a function of the portfolio size for different ages of occurrence for the risk of systematic deviations.

4 Numerical application

The data come from a portfolio of LTC heavy claimants of a French insurance company. The period of observation stretches from 01/01/1998 to 31/12/2010. The data are composed of a mixture of heavy disabilities. The pathologies are composed, among others, by dementia, neurological illness and terminal cancer. The data consist for 2/3 of women and 1/3 of men. We have no exogenous information about the LTC claimants. We observe only the aggregated exposition and number of deaths over two dimensions. These are the age of occurrence v of the pathologies and the duration of the care u. The range of ages of occurrence is $[70-90]$ and the maximum duration of the pathologies is 119 months. Figures 1a and 1b display the observed statistics of the dataset.



(a) Number of deaths. (b) Exposure. (c) fitted probability of death.

FIGURE 1: Number of deaths, exposure and $\widehat{q}_u^{\text{claim}}(x+\upsilon)$ obtained in Tomas and Planchet (2013, Section 3.2).

For the numerical application, we use the fitted surface of the disabled lives mortality obtained in Tomas and Planchet (2013). The fitted disabled lives probability $\widehat{q}_u(x+\upsilon)$, displayed in Figure 1c, have been obtained by fitting the adaptive local bivariate likelihood model with local bandwidth factors methodology. This method allows an adjustment based on the reliability of the data to take into account the nature of the risk and varies the amount of smoothing in a location dependent manner. The approach is an extension of the adaptive kernel methods proposed by Gavin *et al.* (1995) to adaptive local kernel-weighted log-likelihoods techniques. For regions in which the exposure is large, a low value for the bandwidth results in an estimate that more closely reflects the crude rates. On the other hand, for regions in which the exposure is small, such as long duration, a higher value for the bandwidth allows the estimate to progress more smoothly.
In addition, we have at our disposal the disability law ϱ_x provided by the same company and we used the periodic mortality tables TH00-02 and TF00-02 provided by the French Institute of Statistic (INSEE) to compute the mortality law q_x^{ins} of the insured population. The computations are carried out with the help of the software R, R Development Core Team (2014).

We apply model (5) and vary σ_ϵ from 1 to 20 %. The remaining life expectancy of an insured individual varies slightly with σ_ϵ, being around 3.02 months for an insured individual aged 80. We then measure the impact of uncertainty on the expected disabled lifetime of a policyholder by computing the relative difference δ between the 95 % quantile of the simulated remaining lifetime from $K = 5000$ simulations and its expectation (see equation (6)).

Figure 2 shows the impact of uncertainty on the remaining life expectancy. The impact of uncertainty is relatively linear on the remaining life expectancy for a given age. However, it is increasing with the age as the remaining disabled lifetime of a policyholder is decreasing with the age, which in turn induces an increase of the uncertainty.

Experts consider an uncertainty associated to the remaining disabled lifetime of a policyholder of $\pm$ 5 months at all ages of occurrence. It means, for instance, that the remaining disabled lifetime when the disability occurred at age 80 varies by approximatively 1.36 %. In addition, its variation increases greatly at the highest age of occurrence. In order to be coherent with the experts' opinion, we look at which level of volatility σ_ϵ leads to a resulting uncertainty δ corresponding to the variation observed by the experts. The results are displayed in Table 1.

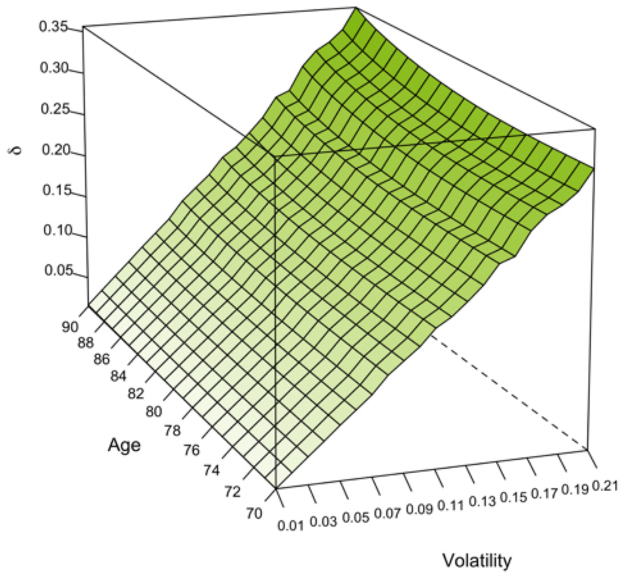


FIGURE 2: Relative difference δ between 95 % quantile of the simulated remaining disabled lifetime and its expectation.

Age of the policyholder	Remaining disabled lifetime	Variation Experts' opinion (%)	Corresponding σ_ϵ (%)
70	4.16	0.91	9.33
71	4.12	0.92	9.33
72	4.06	0.93	9.33
73	4.00	0.95	9.33
74	3.92	0.97	9.33
75	3.82	0.99	9.33
76	3.71	1.02	9.44
77	3.58	1.06	9.44
78	3.43	1.10	9.44
79	3.23	1.17	9.56
80	3.01	1.25	9.58
81	2.78	1.36	9.67
82	2.52	1.50	9.96
83	2.24	1.68	9.96
84	1.95	1.93	9.96
85	1.65	2.29	10.05
86	1.35	2.80	10.05
87	1.05	3.61	10.06
88	0.75	5.02	10.29
89	0.48	7.95	10.52
90	0.22	17.07	10.52

TABLE 1: Remaining disabled lifetime of a policyholder (in months), variation according experts' opinion and corresponding level of volatility σ_ϵ.

We then compute the ratio between the 99.5 % quantile of the simulated sum of the discounted cash flows and the best estimate of the reserve for further levels of volatility in Figure 3 (color lines). We also obtain the corresponding ratio when varying the volatility with the age of occurrence of the disability according the experts' opinion (red dotted line) and compare the results of the QIS5 longevity shock (black dotted line).

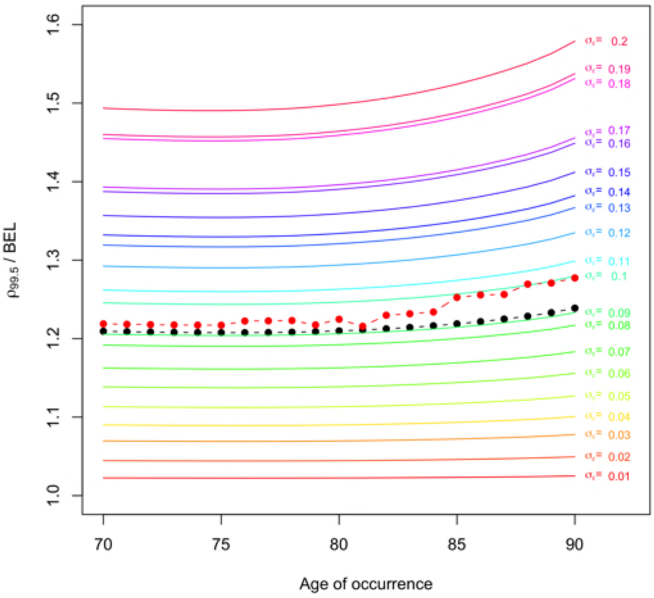


FIGURE 3: Ratio between the 99.5 % quantile of the simulated sum of the discounted cash flows and the best estimate of the reserve.

With a fixed level of volatility $\sigma_\epsilon = 9\,\%$, the ratio between the quantile at 99.5 % of the simulated sum of the discounted cash flows and the best estimate of the reserve is around 20.6 % at age 80, i.e. the capital required for covering the uncertainty arising from the systematic deviations is approximatively 20.6 % of the best estimate.

In applying a reduction of 20 % on the disabled lives death probabilities with the same logic as the longevity shock described in the QIS5 specifications, the remaining disabled life expectancy of a policy-holder aged 80 increases from 3.01 to 3.64 months, meaning a gain of 20.9 %. In consequence, fixing the volatility of the disturbance at 9 % appears to be relatively coherent with the calibration of the standard formula as illustrated in Figure 3, black dotted line.
However, varying the volatility with the age of occurrence of the disability according the experts' opinion, Figure 3 red dotted lines, seems inconsistent with the shocks described in the QIS5 specifications. The standard formula has been calibrated on *standard* mortality which differs greatly from heavy LTC claimants mortality.

We have applied the methodology to other LTC claimants datasets. It appears that the results are insensitive to the underlying structure of the disabled lives survival law. This is explained by the fact that we are working on the core of the distribution, i.e. the general form of the survival law. In consequence, the underlying structure has no impact. In addition, the model only assesses the risk associated with the uncertainty arising from the systematic deviations to which the risk of random fluctuations must be added.

As mentioned in Section 3.3, computing the quantiles of the distribution of Λ gives a biased evaluation of the SCR. In the following, we use the general approximation proposed by Guibert *et al.* (2010) and compute the ratio between the SCR and the best estimate of the reserve as a function of the portfolio size for different ages of occurrence for the underwriting risk. The result is displayed in Figure 4 with varying the level of volatility σ_ϵ according the age of occurrence of the disability and a cost of capital α of 6 %.

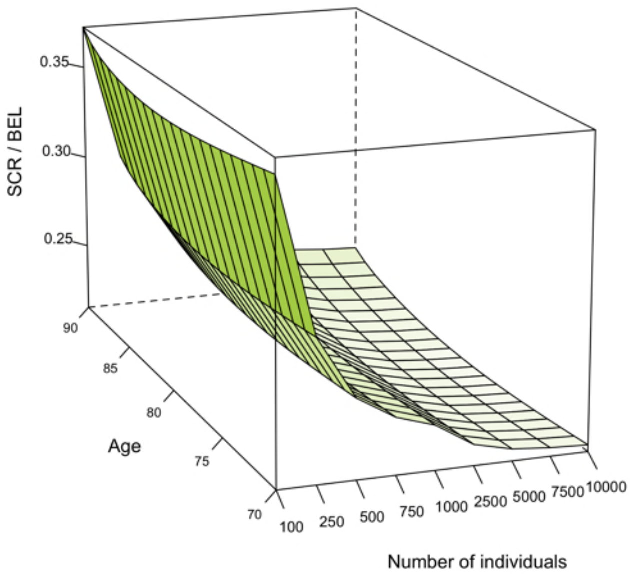


FIGURE 4: Ratio between the SCR and the best estimate of the reserve as a function of the portfolio size.

At age 80, the minimal SCR is around 21.5 % of the best estimate of the reserve for a portfolio of LTC claimants of infinite size, i.e. when ignoring the risk of random fluctuations. For a size of 100 LTC claimants, when random fluctuations are important, the minimal SCR is 35.3 % of the best estimate of the reserve.

We have applied this model to other datasets. Unlike computing the quantiles of the distribution of Λ, the results obtained with the general approximation proposed by Guibert *et al.* (2010) are very sensitive to the choice of the underlying disabled lives survival law. It highlights the impact of the structure of the survival law on the underwriting risk, in particular the importance of the tail of distribution.

5 Conclusions

The uncertainty associated to the underlying disabled lives survival law has important consequences in terms of volatility of reserves.

As soon as we are able to model this uncertainty, the general framework described in Guibert *et al.* (2010) allows us to assess the adequacy of the standard shocks described in the QIS5 specifications with a risk analysis specific to the insurer depending on the structure of the portfolio. We can then build a stochastic model taking into account the constraint of quantifying the uncertainty in a finite horizon and the effect of the risk margin. In this framework, the use of simulations is limited and occurs independently of the portfolio size, as in a *semi-analytical* model, and the computation time is limited.

These reflections highlight the essential assessment of uncertainty associated to the underlying disabled lives survival law as the milestone for a thorough evaluation of the insurer solvency. It leads to consider the implementation of a partial internal model for the underwriting risk.

In addition, the approach presented shows that the level of underwriting SCR obtained is strongly associated to the precision of the assessment of the underlying disabled lives survival law and in particular to the tail of distribution.

Finally, LTC insurance has the particularity to cumulate the effect of long term liabilities with the consideration of future premiums over a long period. The levels of SCR obtained in this study are *true* for a single premium. For a regular premiums contract, these are upper bounds, because the insurer has the option to adjust the price in the future, which is not included in our approach.

Acknowledgment

The authors wish to thank Q. Guibert and F. Lusson for helpful and constructive suggestions which they provided in relation to an earlier draft of this article. We are also grateful to an anonymous referee for a careful reading of the manuscript and comments which led to an improved version of the paper.

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