Best Estimate Calculations of Savings Contracts by Closed Formulas

Version 0.1

François BONNIN
francois.bonnin@primact.fr
Marc JUILLARD
marc_juillard@yahoo.fr
Frédéric PLANCHET
frederic@planchet.net

June 2013

Prim'Act

Introduction

One of the major difficulties of the implementation of Solvency 2 in life insurance is the calculation of the value of best estimate liabilities (fair value) for participating contracts.

Practitioners are turning to *ad hoc* approaches by projecting the flow of benefits under the contract with Markov models, and obtaining numerical results relies heavily on simulation. If it helps describe the flow dynamics accurately, cumbersome calculations make these models difficult to use, configure and maintain. In particular, the use of these approaches within the framework of internal models is particularly difficult (*cf*. Bauer *et al*. [2010]).

Markov style models mentioned above are poorly suited to ORSA projections, because of the large computation time needed and the lack of robustness (which is mainly due to over parameterization).

Thus, our goal in this paper is to build a model able to take into account complex contracts for computing projected best estimates valuations well suited to the ORSA framework.

Introduction

To achieve this goal, we develop a simple framework to compute a coefficient (with a closed formula) which when applied to the mathematical reserve gives the associated fair value of the contract.

Indeed, in general we observe that the best estimate value is near the mathematical reserve (between 95% and 105% of it on most cases). Thus we seek a coefficient to be applied to the mathematical reserve that accounts for the time value of options.

For the Solvency Capital Requirement (SCR) calculation and projection, we adapt here the model described in Guibert *et al*. [2012] to life insurance. The framework is built by directly specifying the dynamics of the increase rate of the contract. In our model the best estimate value of the contract becomes computable and its application in the ORSA framework shows all its interest.

In particular we obtain an explicit expression of the SCR which is easily computable using basic simulation technique.

AGENDA

1. **Basic Framework**
2. Balance Sheet Modeling and SCR Computation
3. Numerical Application

1. Basic Framework

Consider a savings contract with a surrender value for a policyholder that evolves according to (we denote by t=0 the calculation date)

$$VR(t) = VR(0) \times \exp\left(\int_0^t r_s(u)\,du \right)$$

the value of the mathematical reserve at time *t* is

$$PM(t) = VR(t) \times \exp\left(-\int_0^t \mu(u)\,du \right)$$

For the current contract, the payment of the mathematical reserve in case of early withdrawal (ratchet or death) and the term *T* of the contract, assumed to be fixed (non-random), both determine the benefits of the contract. The flow of updated service contract considered here is simply expressed as a function of τ, the release date (random) of the contract (which is the surrender or death time)

$$\Lambda = VR(\tau \wedge T) \times \delta(\tau \wedge T) \qquad \delta(t) = \exp\left(-\int_0^t r(u)\,du \right)$$

1. Basic Framework

The main idea of this paper is to consider that the accumulation rate is affected by two kinds of randomness:

- An hedgeable hazard linked with the market price of the assets;

- Corrections to this return by piloting the accounting result. On this point, even if the management actions are deterministic, we can consider that there is a source of randomness (not hedgeable) associated with the moment the unrealized profit and loss are booked. Indeed, the book yield of a transfer of assets depends on the market price of the asset but also its cost. This second source of randomness must be introduced into the model.

The proposed model is also the following:

$$r_s(t) = r(t) + \omega(t)$$

with the short interest rate *r* the hedgeable part of risk and ω the non-hedgeable one.

1. Basic Framework

By definition, the best estimate at time *t*=0 of the contract is calculated *via*

$$BEL(0,T) = E^{P^{nh} \otimes Q^h}(\Lambda)$$

with the historical probability P^{nh} modeling the non-headgeable risks and Q^h a risk-neutral probability modeling hedgeable risk (see Planchet *et al*. [2011] or Gerber [1997] for the justification of this formula). Because of the decomposition

$$r_s(t) = r(t) + \omega(t)$$

we assume that we can split the probability (which represents the risk associated with ω) between two components, $P^{nh} = P^i \otimes P^\omega$.

In this decomposition, P^i is associated with usual insurance risks, mostly mutualizable ones (mortality, structural ratchet, *etc*.) and P^ω stands for the risks associated with ω.

We assume that usual insurance risks (P^i) and other risks ($P^\omega \otimes Q^h$) are independent.

1. Basic Framework

The best estimate of the contract is

$$BEL(0,T) = E^{P^{\omega} \otimes Q^{h}}\left(BEL^{F}(0,T)\right)$$

with $BEL^{F}(0,T) = E^{P^{i}}\left(\Lambda \middle| F\right)$. Conditioning to *F* means conditioning to financial risk (that is risk that affects the return of the contract, hedgeable or not).

At this stage, we need to make an assumption about ω and μ to obtain an explicit formulae.

The process ω is modeled by an Ornstein-Uhlenbeck process

$$d\omega(t) = k \times \left(\omega_{\infty} - \omega(t)\right)dt + \sigma_{\omega} dB(t)$$

The market often retains a target rate of revalorization close to the risk-free rate (TME, 10-year OAT, *etc*.); this fact motivates our choice. Moreover, the revalorization rate by the contract is determined by the return on assets (its expectation equals to the risk-free rate under a risk-neutral probability) and also smoothing mechanisms induced by accounting principles.

1. Basic Framework

Assume now that the surrender μ is decomposed into the sum of a structural (idiosyncratic) and a cyclical component

$$\mu(u) = \mu_i(u) + \mu_c(\omega(u))$$

Under these hypothesis, it can ben shown that the best estimate is

$$BEL(0,T) = VR(0) \times \left(\int_0^T S_i(t)\left(\mu_i(t) \times \theta_1(t) + \theta_2(t)\right)dt + S_i(T) \times \theta_1(T) \right)$$

S_i is given, and θ_1 and θ_2 have explicit form. In applications we will use the discrete form

$$BEL(0,T) = PM(0) \times \left(\sum_{u=1}^{T} \frac{l_{u-1}}{l_0} \times \left(q_{u-1} \times \theta_1(\omega(0),u) + \theta_2(\omega(0),u) \right) + \frac{l_T}{l_0} \times \theta_1(\omega(0),T) \right)$$

We can now use this expression to project the balance sheet.

AGENDA

1. Basic Framework
2. **Balance Sheet Modeling and SCR Computation**
3. Numerical Application

2. Balance Sheet Modeling and SCR Computation

Here we use the formula set out above to calculate the best estimate from the mathematical reserve. For this we use the Markovian character of ω. Then at time *t* we have

$$BEL(t,T) = \rho(t,T) \times PM(t)$$

$$\rho(t,T) = \int_t^T S_{i,t}(u)\left(\mu_i(u) \times \theta_1\left(\omega(t), u-t\right) + \theta_2\left(\omega(t), u-t\right)\right)du + S_{i,t}(T) \times \theta_1\left(\omega(t), T-t\right)$$

$$\rho(t,T) = \rho(t,T,\omega(t)) = \sum_{u=t+1}^{T} \frac{l_{u-1}}{l_t} \times \left(q_{u-1} \times \theta_1\left(\omega(t), u-t\right) + \theta_2\left(\omega(t), u-t\right)\right)$$

$$+ \frac{l_T}{l_t} \times \theta_1\left(\omega(t), T-t\right)$$

2. Balance Sheet Modeling and SCR Computation

We choose the following dynamics for the risk factors (under the historical probability)

$$dr(t) = k_r \times \left(r_\infty - r(t)\right)dt + \sigma_r dB_r(t)$$

$$dr_A(t) = \mu_A dt + \rho\sigma_A dB_r(t) + \sqrt{1-\rho^2}\sigma_A dB_A(t)$$

$$d\omega(t) = k_\omega \times \left(\omega_\infty - \omega(t)\right)dt + \frac{\rho_{s,a}\sigma_\omega}{\sqrt{1-\rho^2}} dB_A(t) + \sqrt{\frac{1-\rho_{s,a}^2-\rho^2}{1-\rho^2}}\sigma_\omega dB_\omega(t)$$

Here we describe the dynamics of the asset value *A* and cash flows of benefits *F*.

$$A(t+1) = A(t)\exp\left(\mu_A + \rho\sigma_A\left(B_r(t+1) - B_r(t)\right) + \sqrt{1-\rho^2}\sigma_A\left(B_A(t+1) - B_A(t)\right)\right) - F(t+1)$$

2. Balance Sheet Modeling and SCR Computation

One can show that

$$F(t+1) \approx VR(t) \times \frac{l_{i,t}}{l_{i,0}} \times \exp\left(\sum_{u=0}^{t-1} \eta\omega(u)\right) \times \left(1-\left(1-q_i(t)\right) \times e^{\eta\times\omega(t)}\right)$$
$$\approx PM(t) \times \left(1-\left(1-q_i(t)\right) \times e^{\eta\times\omega(t)}\right)$$

$$A(t+1) \approx A(t) \times \exp\left(\mu_A + \rho\sigma_A\varepsilon_r(t+1) + \sqrt{1-\rho^2}\sigma_A\varepsilon_A(t+1)\right)$$
$$- PM(t) \times \left(1-\left(1-q_i(t)\right) \times e^{\eta\times\omega(t)}\right)$$

$$PM(t+1) = PM(t) \times \exp\left(\int_t^{t+1} \left(r(u)+\omega(u)-\mu(u)\right)du\right)$$
$$\approx PM(t) \times \exp\left(r(t)+(1+\eta)\omega(t)\right) \times \left(1-q_i(t)\right)$$

2. Balance Sheet Modeling and SCR Computation

The own fund have the following expression

$$E_{t+1} = e^{r(t)} \times \left(\begin{array}{l} A(t) \times e^{\mu_A - r(t) + \rho\sigma_A\varepsilon_r(t+1) + \sqrt{1-\rho^2}\sigma_A\varepsilon_A(t+1)} \\ \quad - PM(t) \times e^{-r(t)} \times \left(1 - \left(1 - q_i(t)\right) \times e^{\eta \times \omega(t)}\right) \\ \quad - PM(t) \times e^{(1+\eta)\omega(t)} \times \left(1 - q_i(t)\right) \times \rho\left(t+1, T, \omega(t+1)\right) \end{array} \right)$$

and we deduce that the SCR is

$$SCR_t = E_t - VaR_t\left(E_{t+1} \times e^{-\int_t^{t+1} r(u)du} ; 0.5\% \right)$$

$$SCR_t \approx E_t - VaR_t \left(\begin{array}{l} A(t) \times e^{\mu_A - r(t) + \rho\sigma_A\varepsilon_r(t+1) + \sqrt{1-\rho^2}\sigma_A\varepsilon_A(t+1)} \\ \quad - PM(t) \times e^{-r(t)} \times \left(1 - \left(1 - q_i(t)\right) \times e^{\eta \times \omega(t)}\right) \\ \quad - PM(t) \times e^{(1+\eta)\omega(t)} \times \left(1 - q_i(t)\right) \times \rho\left(t+1, T, \omega(t+1)\right) \end{array} \right)$$

AGENDA

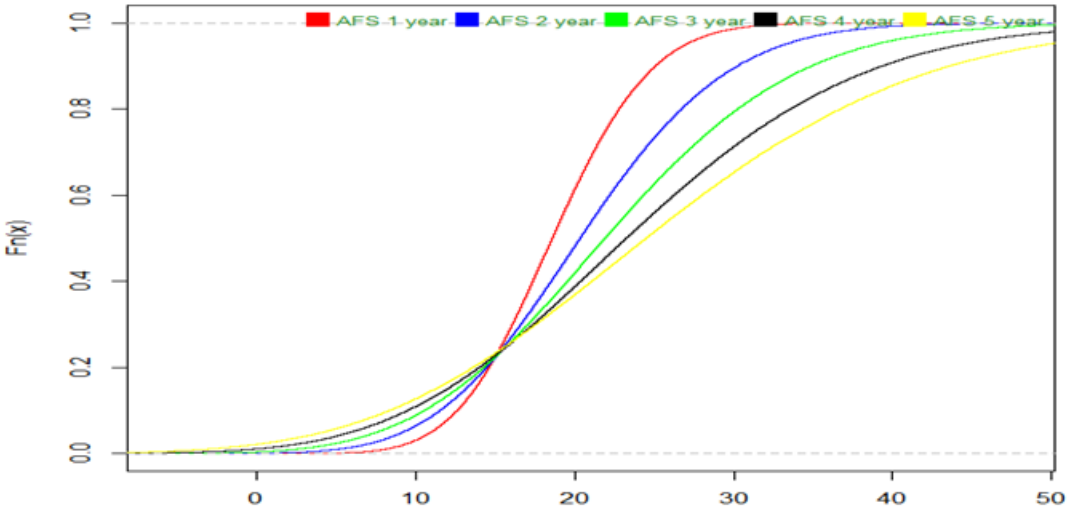
1. Basic Framework
2. Balance Sheet Modeling and SCR Computation
3. **Numerical Application**

3. Numerical Application

The parameters used for the asset allocation are equivalent to consider a 12% equity and 88% bonds allocation. On this basis the projection of the following variables is performed over the next 5 years:

- the value of the mathematical reserve;
- the benefit stream;
- the market value of the assets;
- the simulated paths of financials variables (this information is required for the ORSA process).

At last, this allows projecting the evolution over the next 5 years of the Available Financial Surplus (AFS) represented via the following graph



3. Numerical Application

Based on the distribution of the balance sheet of the company over the next 5 years, we put in place an ORSA process. To do this, we retain an annual 5% quantile. This leads to empirically estimate two quantities:

- The empirical quantile of the AFS for every year j of the next 5 years;

- SCR value associated with each quantile.

The estimation is done in three steps:

- Based on the knowledge of the dynamics of the interest variables we simulate 10 000 realizations of the balance sheet over the next five year ;

- Based on the knowledge of the distribution of the balance sheet relative to the jth year, we select the trajectory corresponding to the empirical quantile;

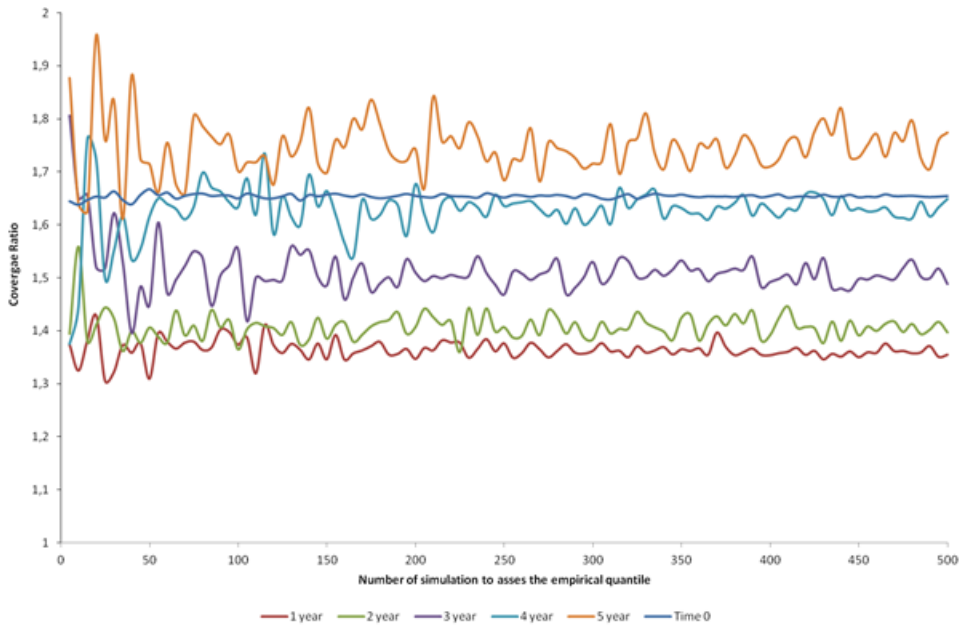
- Conditionally on the information on the selected path, we calculate the empirical quantile at 0.5% of the AFS for the year j+1. This provides the SCR associated with the trajectory withholding for the j^{th} year. We then deduce the quantile coverage ratio of the j^{th} year.

3. Numerical Application

Withholding a quantile based solely on the value of AFS leads to unstable results. The AFS is in fact the imperfect synthesis of the two main variables of interest, the assets and liabilities. Therefore, the three steps above are followed a hundred times, and ultimately we compute the empirical mean of the different simulated quantiles. Hereby the results we find:

Time 0	1 year	2 years	3 years	4 years	5 years
165 %	137 %	141 %	150 %	161 %	171 %

The quantile being empirical, we test the convergence of the result by gradually increasing the number of simulations. The following graph reflects this convergence (results are obtained on the basis of 500 simulations for the quantile and 10 000 for the SCR).



Conclusion

Having a closed formula to go from the mathematical reserve to the best estimate evaluation of the reserve improves dramatically the performance of calculations. Being easily reproducible, it facilitates the process of audit and control.

We propose in this work a model based on the idea that a (French) saving contract is mainly non-hedgeable, because of the accounting rules effect on the revalorization rate of the contract. With this observation, the hedgeable part of the flows is « absorbed » by the discounting process, which leads to very simple calculations.

Conclusion

This approach models the behavior of the insurer with a parameter k - representative of its ability to react to the market - and that of the insured with a parameter η – representing its responsiveness. We can make an implicit computation of these behavioral parameters. according to the results given by an evaluation as part of a traditional ALM model initially to calibrate the model. This will be the subject of further work.

This approach also provides us a powerful tool for making projections of SCR along a « critical path ». This is especially interesting when seen as part of an ORSA process, like time dependent stress scenario analysis

This first analytical framework can then be expanded to capture more complex effects, such as the wealth effect of the insurer through its management of unrealized losses. This will be the subject of future work to jointly model the book value and market value of assets.

References

AASE K.K., PERSSON S.A. [1996] « Valuation of the Minimum Guaranteed Return Embedded in Life Insurance Products », Wharton School Working Paper, 96-20.

BACINELLO A.R. [2003] « Fair Valuation of the surrender option embedded in a guarantee life insurance participating policy », Trieste University Working Paper.

BALLOTTA L. [2004] « Alternative framework for the fair valuation of participating life insurance contracts », Cass Business School, Actuarial Research Paper n°157.

BAUER D., BERGMANN D., REUSS A. [2010] « Solvency II and Nested Simulations – a Least-Squares Monte Carlo Approach », Proceedings of the 2010 ICA congress.

BONNIN F., PLANCHET F., JUILLARD M. [2010] « Applications de techniques stochastiques pour l'analyse prospective de l'impact comptable du risque de taux. », Bulletin Français d'Actuariat, vol. 11, n°21.

BRIGO D., MERCURIO F. [2006] Interest Rate Models - Theory and Practice. 2nd Edition. Springer.

BRIYS E., de VARENNE F. [1994] « Life insurance in a contingent claim framework: pricing and regulatory implications », The Geneva Papers on Risk and Insurance Theory 19, 53-72.

CAJA A., PLANCHET F. [2010] « La mesure du prix de marché du risque : quels outils pour une utilisation dans les modèles en assurance ? », Assurances et gestion des risques, Vol.78 (3/4).

CEIOPS [2010] Technical Specifications for QIS5, European Commission.

FILIPOVIC D. [2009] Term-Structure Models. Springer.

GERBER H. [1997] Life Insurance Mathematics, Third Edition, Springer.

GUIBERT Q., JUILLARD M., PLANCHET F. [2012] « Measuring uncertainty of solvency coverage ratio in ORSA for Non-Life Insurance », European Actuarial Journal, To appear.

HAINAUT D. [2009] « Profit sharing: a stochastic control approach », Bulletin Français d'Actuariat, vol. 9, n°18.

PLANCHET F., LEROY G. [2011] « Solvabilité 2 : quels standards pour le risque de marché ? », la Tribune de l'Assurance (rubrique « le mot de l'actuaire »), n°156 du 01/03/2011.

PLANCHET F., THEROND P.E., JUILLARD M. [2011] Modèles financiers en assurance. Analyses de risque dynamiques - seconde édition, Paris : Economica.

R DEVELOPMENT CORE TEAM [2012] R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria. URL: http://www.R-project.org.

Contacts

Frédéric PLANCHET
frederic@planchet.net

François BONNIN
francois.bonnin@primact.fr

Marc JUILLARD
Marc_juillard@yahoo.fr

Prim'Act
42 avenue de la Grande Armée
F - 75017 Paris
+33-1-42-22-11-00

ISFA
50 avenue Tony Garnier
F - 69007 Lyon
+33-4-37-38-74-37

http://www.primact.fr
http://www.ressources-actuarielles.net
http://blog. ressources-actuarielles.net