Comparing Micro- and Macro-Level Loss Reserving Models *

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Abstract

Accurate loss reserves are essential for insurers to maintain adequate capital and to efficiently price their insurance products. Loss reserving for Property & Casualty insurance is usually based on macro-level models with aggregate data in a run-off triangle. The macro-level models may generate material errors in the reserve estimates when assumptions underlying the estimates evolve over time in an unanticipated way. In recent years, a small set of literature has proposed reserving models that use underlying individual claims data to estimate outstanding liabilities based on individual claim level information, analogous to approaches used in the life insurance industry. These models are referred to as "micro-level models". In this study, we specify a micro-level model with a hierarchical structure to model the individual claim development that has the flexibility to accommodate assumptions that evolve dynamically over time. To assess the performance of this model, we simulate claims data under different environmental changes and use both the macro- and micro-level models to estimate the outstanding liabilities. The performance of the models is evaluated by comparing the predictive distributions of the reserve estimates. The results demonstrate that there are many scenarios in which the micro-level model outperforms the macro-level model by generating reserve estimates with smaller reserve errors and higher precision. For actuaries responsible for setting reserves, this study highlights scenarios in which micro-level models outperform traditional macro-level models and so can provide a new tool to provide insights when establishing accurate loss reserves.

*Keywords: Micro-level loss reserving, hierarchical model, simulation

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1 Introduction

In order to provide for future claim liabilities, insurance companies need to set up loss reserves. A loss reserve represents an insurer's estimate of its outstanding liabilities for claims that occurred on or before a valuation date. As loss reserves appear in insurers' balance sheets and financial statements as the largest liability, accurately estimating the outstanding claims liabilities is extremely important for insurers. Under-reserving may result in failure to meet liabilities and even insolvency of the insurers. Conversely, an insurer with excessive reserves may show a weaker financial position than it truly has and lose its market share. Reserves also provide an estimate for the cost of insurance. Insurers always need to refer to their reserves when they try to determine whether pricing changes are needed in the rate-making practice. An inadequate reserve may lead to the conclusion that pricing is adequate when it is not. On the contrary, reserve estimates that are too high may result in overpricing, limiting the insurer's growth opportunities and weakening its competitive position in market. Loss reserves are usually set by actuaries. In the U.S., a statement of actuarial opinion regarding loss and loss adjustment expense reserves must accompany insurers' annual statements. Hence, it is an obligation for actuaries to develop reserving models that can generate reserve estimates with better quality.

To illustrate, Figure 1 shows the development process of a typical P&C or health insurance claim. A claim that occurs at time T is reported to the insurer at time W, then one or several transactions follow to make payments for the claim until the settlement at time S. The gap between occurrence and reporting, U, is called "reporting delay", and the gap between reporting and settlement, SD, is called "settlement delay". Insurer values the portfolio periodically. The claim is an incurred but not reported (IBNR) claim at valuation date τ_1; a reported but not settled (RBNS) claim at valuation date τ_2; and a settled claim at valuation date τ_3. At the first two valuation dates, the claim has a non-zero outstanding liability that must be estimated. For many lines of insurance business, the development of insurance claims can be long, requiring insurers to establish loss reserves to provide for future claim liabilities.

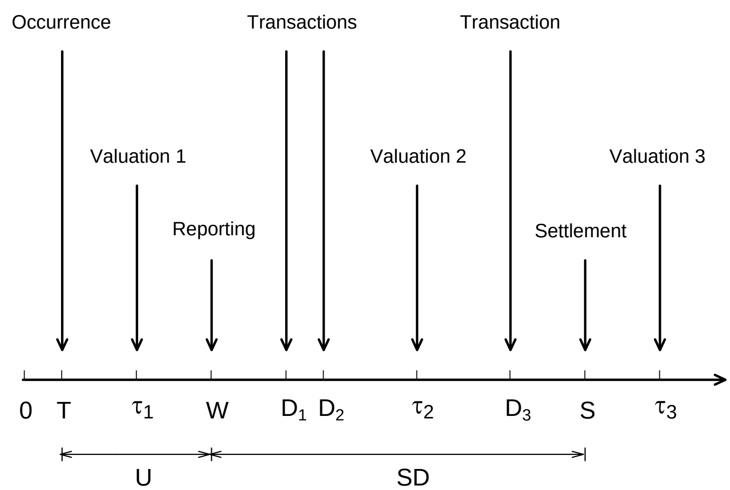

Figure 1: Development of a Property and Casualty Claim. The claim occurs at time T and is reported to the insurer at time W. Multiple transactions occur at D_1, D_2 and D_3. The claim is settled at time S, and τ_1, τ_2 and τ_3 are three possible valuation dates. Further, U is the reporting delay and SD is the settlement delay.

1.1 Traditional Loss Reserving Methods

Loss reserving for insurance is traditionally based on aggregate data in a run-off loss triangle. Among those traditional methods (referred to as "macro-level models"), the chain-ladder technique is the most widely used one. The key assumption of the chain-ladder technique is that claims recorded to date will continue to develop in a similar manner in the future. However, in many practices, there are significant changes in the environment, which could violate this assumption and bias the reserve estimates generated by chain-ladder approach. An *environmental change* refers to a change in the insurer's internal management and operation, or a change in the external business, economic, and legal environment. Commonly seen environmental changes include changes in product mix, benefit level, regulation, inflation, and claims adjusting system, etc. Actuaries sometimes use so-called "trending" techniques to handle environmental changes. "Trending" refers to estimating the impact of environmental changes with a trend rate over accident years implied by the aggregate data, and modifying the loss development projection according to the estimated trend rate. In practice, trending is an ad hoc activity that highly depends on actuarial judgment and the on-going environmental changes. Nevertheless, there are limitations in the use of trending techniques, e.g., a typical "trending" procedure only estimates a constant trend rate for the differences in claims amounts or counts over accident period direction. We will see that these "trending" techniques are usually not as flexible or responsive as needed to fully capture the changes in the environment.

Another commonly-used macro-level reserving method is the expected claims technique. It projects the ultimate claims based on actuaries' prior estimates rather than the claims experience. Other macro-level models, such as Bornhuetter-Ferguson (B-F) method and Cape-Cod method, are constructed as a blend of the chain-ladder and the expected claims techniques (Friedland 2010). By definition, these methods are able to deal with environmental changes by using appropriate prior estimates for ultimate claims. However, when the environment undergoes many rapid and complex changes, it may be questionable to assume that actuaries' expectations are reliable to reflect the impact from the environment. Under these circumstances, methods with prior estimates may also generate material errors in the reserve estimates.

A strength and limitation shared by all macro-level models is that they are based on aggregate data found in a run-off triangle. This is a strength in that the reserve estimates are simple to calculate and interpret. It is a limitation in that aggregate methods are not designed to adapt to rapidly changing environments. Prediction errors given by macro-level models can be disappointingly large (England and Verall 2002), largely due to the small set of data available in the triangles. Lack of robustness and over-parameterization are also issues with macro models due to the effect of a small data set.

While insurance companies always had access to extensive micro-level data, computational limitations have traditionally prevented their use. The traditional reserving methods were adopted because of their simplicity. At present, insurance practitioners certainly have the ability to perform more rigorous reserving models with micro-level information, but traditional methods are still dominant in loss reserving practice. Researchers and actuaries have started to question the continuing use of aggregate data when the underlying extensive micro-level information is available and the computation is feasible, see, e.g., England and Verrall (2002).

The limitations of macro-level models are primarily due to the inability to use individual claim level development data and other micro-level information in loss reserving. Essentially, aggregation of claims development requires homogeneous claims in the insurance portfolio. When there is a high degree of heterogeneity in the claims development process imposed by either the inherent nature of the claims or changes in the external environment, the aggregation might be questionable and more advanced reserving models are desirable. We highlight several such circumstances in the following paragraphs.

Changes in Product Mix. An insurance portfolio is usually not homogenous, but a mix of claims with different characteristics, and the mix may change over time. If some claim-level characteristics have an impact on the individual claim level development patterns, then the aggregate level claims development patterns recorded in the run-off triangle may change as the product mix changes. This may violate the key assumption of the chain-ladder technique and bias the reserve estimates. The failure of the traditional macro-level reserving methods under a changing product mix is well demonstrated by Friedland (2010). Guszcza and Lommele (2006) illustrates the problem of the basic chain-ladder under a changing product mix with simulated data.

Inflation. Inflation has a great impact on claims cost, especially for long-tail lines of business. Claims escalation is often affected by additional factors other than the general inflation measured by the consumer price index (CPI). For example, auto liability claims are affected by medical costs, litigation costs and wage levels of car repairers. Claims inflation due to these additional factors is referred to as super-imposed inflation. To handle the impact of inflation, an appropriate index function that measures the claims inflation pattern over time is needed to discount the nominal payments. Nevertheless, it is difficult to estimate the index function when super-imposed inflation exists, as the rate is different from CPI and often volatile over time. It is customary to use the trending techniques combined with external information regarding inflation rates to deal with the impact of inflation on the run-off triangle.

Changes in Regulation. Insurance is a highly regulated industry. In the US, insurance regulations and laws usually vary by state, and they are frequently revised. Some regulations directly specify the maximum duration in which a benefit is payable after a claim is reported. Changes in these regulations may have a great impact on the claim development speed. For example, workers compensation indemnity benefits are often available within a maximum compensation period specified by state-level regulations. If the maximum compensation period is shortened by a new regulation, then the claims occurred after the effective date of the new regulation are likely to have a shorter settlement delay or a faster development speed.

Changes in Claims Processing. Insurance companies may experience changes in the internal organization and management due to strategic adjustments or external forces. These changes may have an great impact on the claims processing scheme. For example, an insurer may strengthen its case outstanding review process, which changes the development patterns of the incurred losses; an insurer used to be liberal in paying claims may find itself paying too many unnecessary claims, and decide to be more strict on its claims processing, resulting in lower paid losses in recent years; a new claims adjusting team may adopt a more efficient claims processing scheme and hasten the claims payment process.

1.2 Micro-Level Loss Reserving Models

A small set of academic literature has arisen over the last 20 years that studies micro-level stochastic models (also called individual claim level models) for loss reserving. Unlike traditional macro-level methods, these models use individual claims data as inputs and estimate outstanding liabilities for each individual claim. They capture the *micro-structure of claim development* and use *micro-level covariates*. Here the *micro-structure of claim development* refers to the lifetime development process of each individual claim, including events such as claim occurrence, reporting, payment transactions and settlement; and the *micro-level covariates* refer to covariate information about the policy, policy-holder, claim, claimant, and transactions. A micro-level model often has a hierarchical specification that contains several blocks, each handling a part of the claim development process. For example, a micro-level model could have a block to model the claim occurrence time, a block to model the reporting delay, and another block to model the multiple loss payments.

Well-specified micro-level models are expected to generate reserve estimates with reliable quality. Due to the ability to model individual claim level development and to incorporate micro-level covariate information on the policy-, claim- and transaction-level, micro-level models can efficiently handle heterogeneities in claims data. The large amount of data used in modeling also avoids issues of over-parameterization and lack of robustness. The advantages of micro-level models are especially significant under changing environments, as environmental changes can be indicated by appropriate covariates, and the models' hierarchical nature makes it easy to estimate the impact of these changes on the claims development.

Norberg (1993 and 1999) and Arjas (1989) built a mathematical framework for applying a marked Poisson process in modeling claims development on an individual claim level. Based on this theoretical framework, several groups developed individual claim level loss reserving models and used case studies for illustration, see, e.g., Haastrup and Arjas (1996), Larson (2007), Antonio and Plat (2012), and Pigeon, Antonio and Denuit (2012). Another stream of literature focuses on predicting the number of IBNR claims with marked Poisson processes. Jewell (1989) presented the theoretical framework. Following this framework, Zhou and Wang (2009), and Zhao and Zhou (2010) developed models using a semi-parametric specification and used simulated data for illustration. In sum, we are aware of fewer than 20 research articles on the topic of micro-level reserving. Among them, over a half are either pure theoretical papers or theoretical papers with very brief case studies. Papers that provide detailed and complete implementation of the micro-level models on empirical data are currently lacking in the literature. To our knowledge, Antonio and Plat (2012) and Pigeon, Antonio and Denuit (2012) are the only studies that demonstrate such level of detail. While the existing literature has contributed solid mathematical framework for micro-level reserving models, this paper provides a more practical approach to demonstrate how to implement these models and the benefits that one receives from them.

1.3 Overview of the Present Research

The purpose of this study is to highlight the scenarios in which micro-level models outperform traditional macro-level models by evaluating the performance of both the macro- and micro-level models with simulated data. We also hope to draw more attention from the P&C practitioners by supplementing the existing micro-level research with a more realistic and implementable model.

The advantages of the micro-level models relative to the macro-level models are particularly significant for long-tail lines of business when there are changes in the environment, hence, we focus on the comparison of models for a book of business with a relatively long tail under changing environments. The simulated scenarios include several environmental circumstances that are commonly seen in practice. Here a scenario refers to an environment in which the insurance portfolio of interest is operated. It includes the external business, economic or regulatory environment, and the insurer's internal operation or management environment. A steady environment without any significant changes is first explored as a benchmark. Then different environmental changes (corresponding to changes in product mix, regulation, claims adjusting scheme, and inflation) are imposed by adjusting simulation parameters and using appropriate covariates.

We simulate claims data under different scenarios, and for each simulated dataset, apply various reserving methods to generate reserve estimates. Monte-Carlo techniques are used to obtain distributions of the reserve estimates. The performance of the reserving models is evaluated by comparing the distributions. As the most widely used reserving method, the basic chain-ladder technique is evaluated in each simulated scenario. We also perform a so-called "trended chain-ladder" method in which the "trending" techniques are used to handle the environmental changes. The proposed micro-level model has a hierarchical structure that contains models for five blocks of the claims development: claim occurrence time, claim reporting delay, transaction times, transaction types, and transaction-level payment amounts. The micro-level model is first applied without model risks, and then applied with intentionally imposed model mis-specifications to check the robustness of the model.

The remainder of the paper is organized as follows. In Section 2, the simulation procedure and the scenarios are described. In Section 3, results from each scenario are presented. Section 4 discusses the results and Section 5 concludes the study. Supporting details are in the appendices.

2 Methodology

2.1 Simulation Procedure

For each scenario, the simulation procedure contains four steps: (1) a generation routine that draws the individual claim level full development from a population distribution; (2) an estimation routine that estimates the distribution parameters based on the claims development data that is censored with respect to a valuation date; (3) a prediction routine that projects the claims development after the censoring date and obtains the reserve estimates; and (4) an evaluation routine that compares the distributions of the reserve estimates from different models. The full development of a claim refers to all the events throughout the entire life of a claim, including accident occurrence, claim reporting, multiple transactions, and claim settlement.

The population distribution of the claims development process is explicitly specified with distributional assumptions for five blocks: (1) the accident occurrence times follow a uniform distribution; (2) the reporting delays are assumed to be zero; (3) the transaction occurrence times are governed by a survival model with time-dependent hazard rates; (4) the transaction types are determined by a multinomial logit model; and (5) the transaction-level payment amounts follow a log-normal distribution. The distribution parameters are denoted by $\boldsymbol{\theta}$. Section 1 of Appendix 1 documents the detailed assumptions for the population distribution. In most scenarios (Section 3.1-3.6), we only consider reserving for reported claims, as the reporting delay is assumed to be zero for every claim. In Section 3.7 and Section 5 of Appendix 1, we extend the model to consider both the reported and IBNR claims by assuming the reporting delay follows a Poisson distribution.

The impact of the changing environment is generated by letting the population distribution depend on covariates that may change. Although multiple covariates could be easily incorporated in any block of the population distribution, we specify only one covariate, denoted by X, for the population distribution under each scenario[1]. The "one covariate" assumption simplifies the computation while still allowing us to demonstrate the desirable properties of the micro-level models. This covariate can be a time-constant variable that is observable to the insurer at the time of notification, or a time variable such as accident year (AY), development year (DY) or calendar year (CY). The covariates used in each scenario will be specified in Section 3.

For each scenario, the estimation and prediction routines are based on A samples, each containing 5000 claims[2] with the full development processes, drawn from the population distribution. In most of the analysis, we use $A = 100$. A single iteration of sampling is performed as follows. In the a^{th} iteration, a sample of 5000 claims is drawn from the population distribution. With respect to the valuation date, the actual outstanding liability $R^{(a)}$ for this sample can be computed with the future development. For the micro-level model, we estimate the population distribution parameter $\boldsymbol{\theta}$ with the maximum likelihood method based on the past development of claims in the sample, and let $\hat{\boldsymbol{\theta}}$

[1] Scenario 3 uses two covariates, but both of them are transformed from the same information. Scenario 6 uses multiple covariates to simulate more than one type of environmental changes.

[2] As we will describe later, we simulate an accident period of 10 years, so the number corresponds to 500 claims per year on average. It may represent the number of claims in the line of workers compensation for a small- to medium-sized insurance company. This is based on a dataset extracted from NAIC Schedule P.

denote the MLEs of parameters. The estimation routine is described in detail in Section 3 of Appendix 1. The reserve estimates are obtained through a Monte-Carlo valuation, that is, drawing B pseudo-samples of the future development for the 5000 claims in the sample, from the population distribution with the estimated parameters $\hat{\boldsymbol{\theta}}$. In most of the analysis, we use $B = 100$. With $\hat{R}_b^{(a)}$ denoting the outstanding liability for the b^{th} pseudo-sample, then the reserve estimate for the a^{th} sample of 5000 claims is $\hat{R}^{(a)} = \sum_{b=1}^{B} \hat{R}_b^{(a)}/B$. Details about the prediction routine is documented in Section 4 of Appendix 1. After the prediction routine, we obtain a series of reserve estimates, one for each sample of 5000 claims, denoted by $\hat{R}^{(1)}$, $\hat{R}^{(2)}$,...,$\hat{R}^{(A)}$.

Recall that we use a covariate in the population distribution to generate the environmental changes for each scenario. In the Monte-Carlo procedure, we incorporate the same covariate in the proposed micro-level model, which simulates a real world situation where the insurer successfully incorporates a predictive covariate in the modeling. We are also interested in the performance of the micro-level model with some mis-specifications. So we also build a mis-specified micro-level model by omitting the covariate in the Monte-Carlo procedure. This is analogous to the situation where the insurer fails to use a predictive covariate in modeling the claims development.

Reserve estimates for the basic chain-ladder technique are obtained through a similar procedure. For each sample of 5000 claims, we aggregate the loss data to form a traditional run-off triangle. We then adopt a chain-ladder method with over-dispersed Poisson (ODP) assumption (Renshaw and Verrall (1998)). The ODP parameters are estimated by MLEs based on the aggregate data in the upper triangle, and B pseudo-samples of the lower triangle are drawn from the ODP distribution with MLEs of the parameters. Then a reserve estimate for the sample of 5000 claims is calculated through the Monte-Carlo procedure, and a series of reserve estimates, $\hat{R}^{(1)}$, $\hat{R}^{(2)}$,...,$\hat{R}^{(A)}$, are obtained.

For the "trended" chain-ladder method, we simply apply a deterministic trending algorithm to get $\hat{R}^{(a)}$ for the a^{th} sample of 5000 claims, i.e., the Monte-Carlo procedure is not used here. Detailed trending procedures are described in Appendix 3 on a scenario-by-scenario basis.

After the generation, estimation, and prediction steps, a series of reserve estimates, $\hat{R}^{(1)}$, $\hat{R}^{(2)}$,...,$\hat{R}^{(A)}$, is obtained for each of the four reserving methods that we are considering. The last step is to compare the performance of these methods. Essentially, loss reserving is to estimate the outstanding liability, denoted by R, by a reserve estimate $\hat{R}$, at a given valuation date. The reserve estimate $\hat{R}$ is a function of the past history of the claims development. It is unbiased if $\mathrm{E}[\hat{R}] = \mathrm{E}[R]$. As in England and Verrall (2002), the quality of a reserve estimate can be measured by mean square error of prediction (MSEP), which is defined by $\mathrm{MSEP}(\hat{R}) = E[(\hat{R} - R)^2]$. To evaluate the performance of a reserving model, we will need to estimate $\mathrm{E}[\hat{R}]$, $\mathrm{MSEP}(\hat{R})$ and the distribution of $\hat{R}$. In the evaluation routine, these quantities are estimated based on the empirical distribution of $\hat{R}^{(1)}$,...,$\hat{R}^{(A)}$. That is,

$$\mathrm{E}[\hat{R}] \approx \overline{\hat{R}} = \sum_{a=1}^{A} \hat{R}^{(a)}/A,$$

$$\mathrm{MSEP}(\hat{R}) \approx \sum_{a=1}^{A} (\hat{R}^{(a)} - R^{(a)})^2/A.$$

While $E[\hat{R}]$ and $\mathrm{MSEP}(\hat{R})$ meet the need to compare various reserving models under a given scenario, they are not convenient for comparisons over different scenarios. We thus use an alternative estimate: the percentage reserve error (RE) defined by

$$RE = \frac{\hat{R} - R}{R} \times 100\%.$$

The expected value, MSEP, and standard deviation of the percentage reserve error can also be estimated. In most of our analysis, we will rely on RE rather than $\hat{R}$. Expected values and MSEPs of RE will be used to perform the comparison. A reserve estimate with good quality would have a close-to-zero $\overline{RE}$ and a small $\mathrm{sd}(RE)$. Following the increasing interest in the full distributions of reserve estimates, we also show the estimated distributions of RE (estimated by the empirical distribution of $RE^{(a)}$), and use them to evaluate the models' performance. The procedure to estimate the first two moments of the reserve estimates ($\mathrm{E}[\hat{R}]$ and $\mathrm{MSEP}(\hat{R})$) is similar to Rosenlund (2012); the only difference lies in the method to get the samples. While Rosenlund's samples are bootstrap pseudo-samples drawn from a pool of individual claims, ours are true samples drawn from the underlying population of claims. The strategy of using percentage reserve errors in evaluating the models' performance was used by Stanard (1985), where the comparison of four macro-level models were demonstrated with simulated data.

2.2 Description of Scenarios

Many different scenarios could be generated by adjusting the population distribution parameters and the covariates of interest. The chain-ladder assumption requires similar claims development patterns over accident years. If the environmental change leads to different claims development patterns over accident years, then the chain-ladder assumption is violated and material errors in the reserve estimates may result. We only focus on scenarios where the assumptions underpinning the chain-ladder predictions do not hold that represent commonly encountered situations in actuarial practice. The six scenarios studied are described in Table 1. Details about the covariates and parameters used to represent each scenario are documented in Appendix 2.

The format of the population distribution allows us to separate the impact that the environmental changes make on the transaction-level payment amounts and the claims development speed. Scenario 1 represents a steady environment. Scenarios 2, 4, and 5 simulate environmental changes that influence the claims development speed. We use settlement delay (SD) to measure the claims development speed, that is, claims that develop faster have shorter settlement delays. For each of the three scenarios, we define two statistics SD_1 and SD_2, as described in Table 2, and use $\Delta SD = SD_1 - SD_2$ to measure the impact of the environmental change on the claims development speed (a higher ΔSD represents a greater impact). We generate three cases with increasing ΔSD for each scenario, i.e., $\Delta SD = 5$ months in Case 1; 9 months in Case 2; and 12 months in Case 3. Scenario 3 focuses on changes in the transaction-level payment amounts over calendar years to simulate an environment under inflation. We assume that there is prior knowledge about the type of inflation (steady, jump, or increasing, etc.), whereas the rate of inflation is unknown and needs to be estimated with the claims development history. Scenario 6 simulates a more realistic environment that undergoes both inflation and a changing product mix.

Scenario	Description	Covariate Information
1. Steady	Thers is not any significant changes in the environment.	N/A
2. Changes in Product Mix	A book of workers compensation insurance is written to construction companies and financial services companies. Claims from financial services companies (referred to as Type 2 claims) develop faster than those from construction companies (referred to as Type 1 claims). There are on average 10% Type 2 claims in AY 1 and then the proportion increases by 8% on average for each subsequent AY.	Policy-holder industry (construction or financial services)
3. Inflation	The book of business is exposed to super-imposed inflation (e.g., medical inflation). Three types of inflation are simulated. In Case 1, inflation is at a constant rate of 3% per CY. In Case 2, the inflation rate is 3% for the first five CYs and 10% thereafter. In Case 3, the inflation rate is 2% in CY 1 and then increases by about 1% in each subsequent CY.	Calendar year for each transaction
4. Changes in Regulation	A new regulation goes into effect at the beginning of AY 6. As a result, claims from AY 6-10 develop faster than those from AY 1-5.	Accident year
5. Changes in Claims Processing	A more efficient claims processing scheme is adopted at the beginning of CY 6. As a result, claims develop faster in CY 6 and thereafter.	Calendar year for each transaction
6. Mixed Scenario	A book of workers compensation insurance might be exposed to medical inflation as described in Case 2 of Scenario 3. Claims are different in three characteristics. The first characteristic is the industry of policy-holder as described in Scenario 2, but here a random proportion of claims is from construction companies for each AY. The second characteristic is claimant age, the average of which gets bigger over AYs. Claims from younger claimants develop faster than those from older clamaints. The third characteristic is injury type. Assume only certain types of injuries would receive tratments that are exposed to medical inflation and a random proportion of claims are with these injuries for each AY.	Policy holder industry, claimaint age, injury type

Table 1: Description of Scenarios. A brief description of the covariate information used in each scenario to generate the environmental change is also listed. See Appendix 2 for a more detailed description of covariates.

Scenario	Settlement Delay (SD_1)	Settlement Delay (SD_2)
2	Median settlement delay of Type 1 claims	Median settlement delay of Type 2 claims
4	Median settlement delay of claims that occur before the new regulation goes into effect	Median settlement delay of claims that occur after the new regulation goes into effect
5	Median settlement delay of all claims if the old claims processing scheme had been in use all the time	Median settlement delay of all claims if the new claims processing scheme had been in use all the time

Table 2: Definitions of Statistics SD_1 and SD_2 for Scenarios 2, 4, and 5. The difference in SD_1 and SD_2 is used to measure the impact of an environmental change on the claims development speed.

3 Results

Table 3 summarizes the expected values, standard deviations, and root mean square error of prediction (root of MSEP) of the percentage reserve errors (RE) generated by different reserving models under each scenario. Distributions of RE are shown in Figures 2-7. We now provide an interpretation of Table 3 and Figures 2-7 in the following six subsections.

Scenario	Case	Basic CL			Trended CL			Micro			Micro w/o covariates		
		mean	sd	RMSEP	mean	sd	RMSEP	mean	sd	RMSEP	mean	sd	RMSEP
1. Steady	1	2.0	7.7	7.9				-2.2	5.5	5.9			
2. Changes in Product Mix	1	11.2	9.3	14.9	5.2	21.8	22.3	-1.9	5.5	5.8	6.7	5.9	8.9
	2	22.5	9.4	24.3	-3.2	18.0	18.2	-1.8	4.8	5.1	17.3	5.7	18.2
	3	30.9	8.4	32.0	-8.7	15.1	17.4	0.3	4.4	4.4	25.2	5.5	25.8
3. Inflation	1	0.7	5.4	5.4				-1.5	4.6	5.4	-18.9	3.8	19.3
	2	-8.5	5.8	10.2	3.9	10.3	11.0	-0.9	5.9	6.0	-49.5	2.8	49.6
	3	-17.4	5.3	18.2	35.0	94.8	100.7	0.4	5.9	5.9	-50.7	2.5	50.8
4. Changes in Regulation	1	21.0	9.7	23.1	23.6	25.7	34.8	-1.4	5.8	5.9	15.5	6.3	16.7
	2	46.4	10.4	47.5	21.0	29.3	35.9	-0.8	4.8	4.9	36.1	5.9	36.5
	3	76.5	11.6	77.3	8.1	21.2	22.6	1.1	4.1	4.2	61.8	5.8	62.1
5. Changes in Claims Processing	1	8.8	7.8	11.7	3.4	7.9	8.6	-2.0	3.3	3.9	2.8	3.7	4.6
	2	19.2	10.0	21.7	6.5	9.3	11.3	-2.3	5.0	5.5	10.7	5.5	12.0
	3	32.8	9.0	34.1	9.7	7.6	12.3	-1.7	4.1	4.5	16.6	5.0	17.4
6. Mixed Scenario	1	-23.2	10.0	25.2				-2.2	11.8	12.0			

Table 3: Summary Statistics of Percentage Reserve Error by Scenario. Four prediction methods are evaluated: the basic chain-ladder (Basic CL), chain-ladder with trending techniques (Trended CL), the proposed micro-level model (Micro) and the micro-level model with omitted covariates (micro w/o covariates). Expected values (mean), standard deviations (sd) and root mean square errors of prediction (RMSEP) are shown calculated.

3.1 Scenario 1: Steady Environment

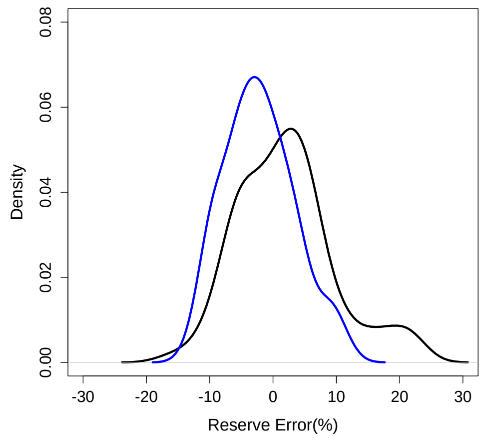

Figure 2: Percentage Reserve Error Distributions under a Steady Environment. The black line shows the result from the basic chain-ladder method and the blue line shows the result from the micro-level model.

Under the steady environment, the population distribution of the claims development is specified in the absence of covariates. Because the environment is steady, no trending is applied to the chain ladder and no covariates are needed for the micro-level model, and so we only compare the basic chain-ladder method and the proposed micro-level model. As shown in Figure 2 and Table 3, both methods perform well. The out-of-sample reserve error distributions are both centered close to 0 and so no material errors in the reserve estimates are observed in either method. Given the relative simplicity of the chain-ladder method, it is remarkable how close the two distributions are to one another.

Nonetheless, the reserve error given by the micro-level model appears to have smaller variation than that given by the basic chain-ladder technique. This difference in the reserving uncertainty is likely to be a result of the amount of information extracted by each model from the claims data. While the chain-ladder technique uses only the aggregate data in the run-off triangle, the micro-level model extracts much more extensive information using the individual claim level information. Although no covariates are used, the information on the individual claims is valuable to allow a closer modeling of loss development, which reduces the reserve uncertainty. This result suggests that the micro-level model is preferable even under a steady environment.

3.2 Scenario 2: Changes in Product Mix

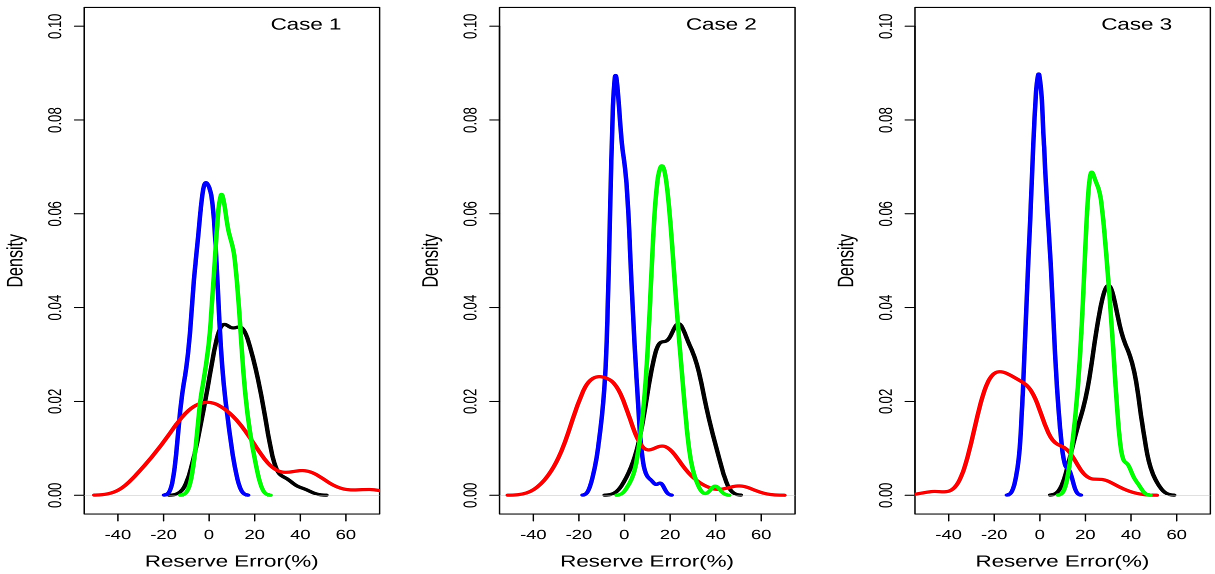

Figure 3: Percentage Reserve Error Distributions by Changing Product Mix Scenario. Black line: the basic chain-ladder; blue line: micro-level model; red line: trended chain-ladder; green line: micro-level model with omitted covariates. The difference in the claims development speed becomes larger going from Case 1 to Case 3.

Under the changing product mix scenario, we introduce the insurer's knowledge of the type of claim (e.g., financial services versus construction worker's compensation) that has impact on the speed of claim development, see, Table 1 for more details.

Figure 3 shows the distributions of the percentage reserve errors under a changing product mix. The basic chain-ladder reserve estimate appears to have positive material error, and the error increases when the change in the product mix becomes larger (going from Case 1 to Case 3). In contrast, the micro-level model (which allows for knowledge of product mix) does not generate material errors in the reserve estimates. This is primarily due to the model's ability to incorporate the claim-level covariate X and to directly estimate the difference in the development speed between the two types of claims. When the covariate is omitted, the micro-level model also generates inaccurate reserve estimates that are biased in the same direction as the basic chain-ladder estimates. It is also shown that the variance of the reserve error given by the micro-level model is much smaller than that given by the basic chain-ladder, suggesting higher precision of the micro-level reserve estimates. According to the result from the trended chain-ladder method, although the trending technique does improve the performance of the chain-ladder in terms of the point estimate, it also brings additional uncertainty to the reserve estimates. We attribute the additional uncertainty to the limitations of the trending technique.

3.3 Scenario 3: Inflation

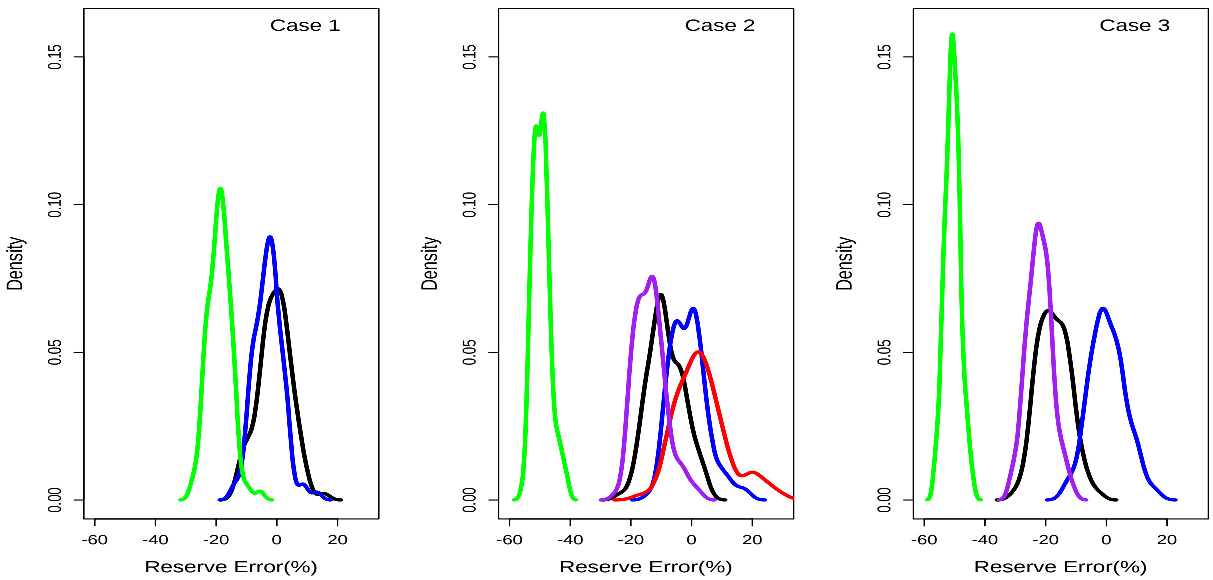

Figure 4: Percentage Reserve Error Distributions by Inflation Scenario. Black line: basic chain-ladder; blue line: micro-level model; red line: trended chain-ladder; green line: micro-level model with both covariates omitted (assuming no inflation); purple line: micro-level model with the second covariate omitted (assuming stable inflation). Case 1: stable inflation of 3% per year; Case 2: inflation rate is 3% in the first five years and 10% thereafter; Case 3: inflation rate is 2% in the first year and increases by about 1% for each subsequent year.

Under the inflation scenario, we simulates three types of inflation: stable inflation in Case 1, inflation with a jump in Case 2, and increasing inflation in Case 3. See Table 1 for more details.

The left panel of Figure 4 shows results under stable inflation. The basic chain-ladder does not generate material error in the reserve estimate. The micro-level model also works well when the covariate is used in the projection, whereas it under-reserves when the covariate is omitted. The reserve errors given by the basic chain-ladder and the micro-level model have comparable variation. A trended chain-ladder is not performed in this case, as the basic chain-ladder works well.

The middle panel of Figure 4 shows the results for Case 2 where a one-time jump in the inflation rate is imposed. The basic chain-ladder technique underestimates the outstanding liability. Trending does help to reduce the material error, but it appears to over-react to inflation and ends up over-reserving. Meanwhile, trending also brings big additional variation to the reserve estimates. In contrast, the reserve estimate given by the micro-level model does not appear to have material errors. When incorrect inflation assumptions are used, the micro-level model under-reserves.

The right panel of Figure 4 shows the results for Case 3 under an increasing inflation rate. The results are similar to Case 2: while the micro-level model gives reserve estimate without material errors, both the basic chain-ladder method and the micro-level model with incorrect inflation assumptions underestimate the outstanding liability. Since the inflation rate has a more complicated time-dependent structure in this case, the material errors are more significant than those in Case 2. The distribution of the trended chain-ladder reserve estimate is not included in the figure, because "trending" does not improve the performance of the chain-ladder method and the variation in the reserve estimate is very big (the standard deviation is over 100%). This is not surprising considering the difficulty in estimating the complicated inflation structure with the limited amount of aggregate data.

The basic-chain ladder method appears to provide some "natural protection" against stable inflation, i.e., the method does not generate material error even if it does not make any adjustments for inflation. A mathematical proof of the "natural protection" can be easily provided by using a flat index function to obtain an inflation adjusted expression for the reserve estimate. Intuitively, stable inflation does not change the claims development pattern over accident years, so the chain-ladder assumption still holds. Nevertheless, this natural protection no longer exists under a more complex inflation structure.

3.4 Scenario 4: Changes in Regulation

Under this scenario, we introduce a regulation revision at the beginning of AY 6 that impacts the speed of claim development. See Table 1 for more details.

Figure 5 shows the distributions of the percentage reserve errors under changes in regulation. The results are similar to those in Scenario 2 under a changing product mix, except that the material errors generated by the chain-ladder technique are larger under this scenario.

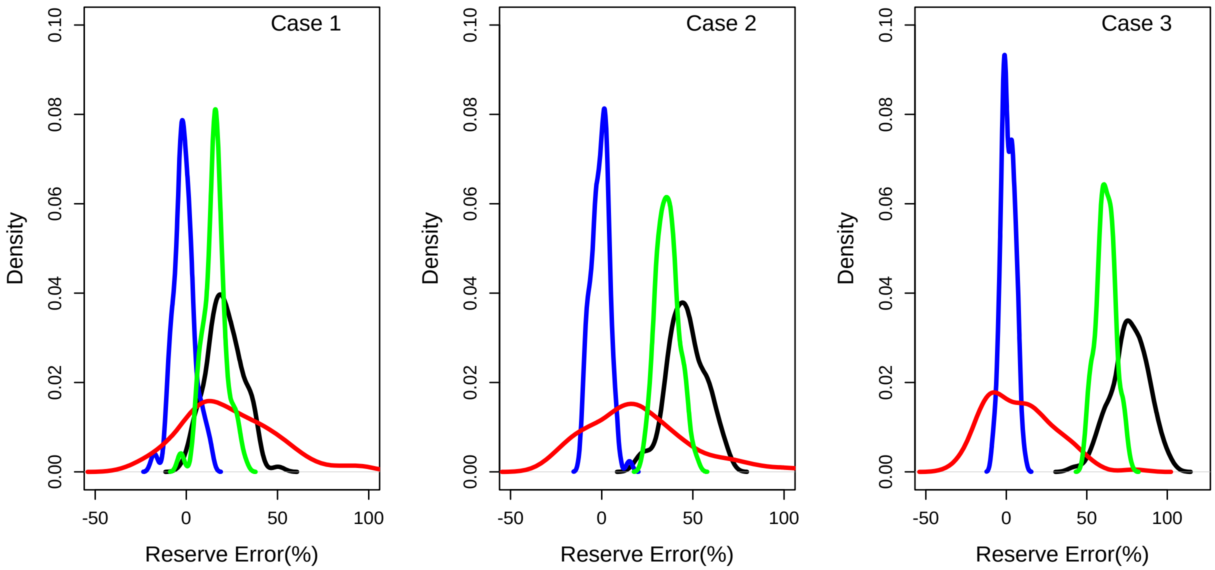

Figure 5: Percentage Reserve Error Distributions by Regulation Scenario. Black line: basic chain-ladder; blue line: micro-level model; red line: trended chain-ladder; green line: micro-level model with omitted covariates. The difference in the claims development speed before and after the regulatory change becomes larger going from Case 1 to Case 3.

3.5 Scenario 5: Changes in Claims Processing

Under this scenario, we introduce a change in claims processing at the beginning of CY 6 that has an impact on the speed of claim development. See Table 1 for more details.

As shown in Figure 6, similar distributions of the percentage reserve errors are observed again, but a comparison with Figure 3 (Changing Product Mix Scenario) and Figure 5 (Regulation Scenario) suggests that the material errors generated by the chain-ladder algorithm are smaller here than those under a changing product mix (Scenario 2) or a regulatory change (Scenario 4).

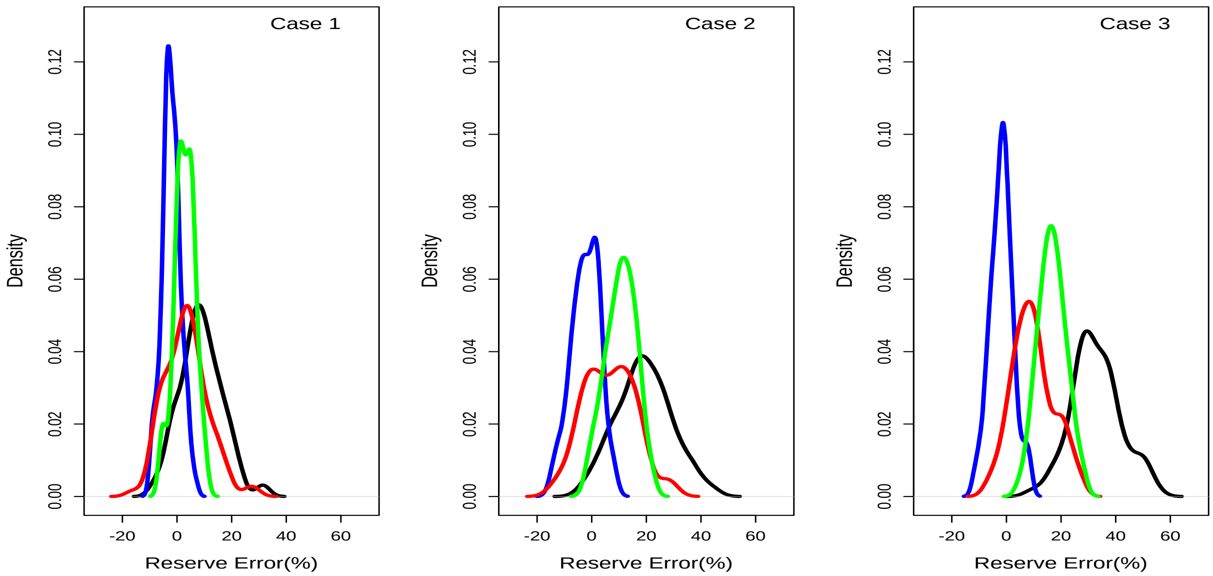

Figure 6: Percentage Reserve Error Distributions by Changing Claims Processing Scenario. Black line: basic chain-ladder; blue line: micro-level model; red line: trended chain-ladder; green line: micro-level model with omitted covariate. The difference in the claims development speed before and after the implementation of the new claims processing scheme becomes larger going from Case 1 to Case 3.

3.6 Scenario 6: Mixed Scenario

The mixed scenario simulates an environment with both inflation and a changing product mix. See Table 1 for more details.

Figure 7 shows the results given by the basic chain-ladder and the micro-level model. The basic chain-ladder under-reserves by more than 20% while the micro-level model does not appear to generate material errors. The variation in the reserve errors is larger than that in the prior scenarios, for many more uncertainties are incorporated in this scenario by generating more than one type of environmental changes. The trending techniques are not applicable to this scenario, because the change in product mix is random and interacted with inflation. It indicates the limitations of the trending techniques when claims are highly heterogeneous and the environmental change can not be approximated by any steady trends.

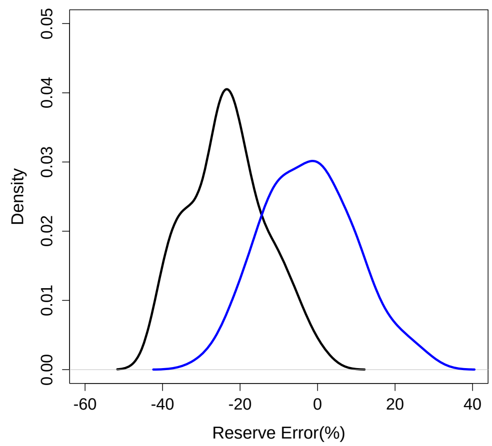

Figure 7: Percentage Reserve Error Distributions by Mixed Scenario. The black line shows the result from the basic chain-ladder method and the blue line shows the result from the micro-level model.

3.7 Modeling with IBNR Claims

We now extend the model to consider both reported and IBNR claims for Scenario 2 (changes in product mix). This is done by relaxing the assumption of zero reporting delays. We start with a simple assumption: the reporting delay follows a Poisson distribution with parameter 1, i.e., the average reporting delay is one month. As shown in Table 4 Panel (a), the results are similar to those under the assumption of zero reporting delays (shown in Table 3). These results assume that the reported process does not depend on claim characteristics.

The characteristics of claims may not only have an impact on the claims development, but also have an impact on the reporting delay. We now incorporate the impact of the covariate X on the reporting delay by letting the Poisson parameter depend on X: the parameter is 2 for Type 1 claims ($X = 1$) and 1 for Type 2 claims ($X = 0$). That is, the average reporting delay for Type 1 claims is two months while that for Type 2 claims is one month. As claim characteristics are known when a claim is first reported, they are unobservable for insurers prior to reporting. Hence, in the projection of IBNR claims, the covariate X is a simulated quantity rather than an observable variable. See Section 5 of Appendix 1 for more details. The results are shown in Table 4 Panel (b). Compared to the results under the assumption of zero reporting delays, now the material errors in the reserve estimates generated by the basic chain-ladder are more significant, whereas the performance of the micro-level model does not change substantially.

Essentially, insurers' observable information differs between reported and IBNR claims and micro-level models's advantage in information usage is not preserved in reserving for IBNR claims. Nevertheless, micro-level models can still be extended to handle IBNR claims by incorporating unobservable factors or error terms in the model. We conjecture that the existence of IBNR claims does not seriously compromise the performance of micro-level models based on the simulation results.

	Case	Basic CL			Trended CL			Micro			Micro w/o covariates		
		mean	**sd**	**RMSEP**	**mean**	**sd**	**RMSEP**	**mean**	**sd**	**RMSEP**	**mean**	**sd**	**RMSEP**
(a)	**1**	9.5	8.0	12.5	2.9	22.7	22.9	-2.3	4.0	4.6	6.4	4.2	7.6
	2	20.0	7.9	21.5	-2.9	18.7	18.9	-1.6	5.0	5.2	15.8	5.9	16.9
	3	29.8	8.6	31.0	-4.0	17.1	17.6	-0.7	4.5	4.5	24.2	5.5	24.8
(b)	**1**	16.4	8.5	18.5	-2.4	21.0	21.1	-2.4	4.9	5.4	6.2	5.4	8.2
	2	27.4	8.7	28.7	-2.6	18.8	19.0	-1.8	5.4	5.7	14.9	6.2	16.1
	3	34.4	8.7	35.5	-7.3	15.8	17.4	-2.1	4.5	5.0	22.3	5.4	22.9

Table 4: Summary Statistics of Percentage Reserve Error by Changing Product Mix Scenario with IBNR Claims. Panel (a): the reporting delay does not depend on the covariate. Panel (b): the reporting delay depends on the covariate. Expected values (mean), standard deviations (sd) and root mean square errors of prediction (RMSEP) are shown.

4 Discussion

4.1 Interpretation of Results

The analysis of the "steady environment" in Section 3.1 shows how well the basic chain-ladder performs under stable conditions. From one viewpoint, this result is fascinating because the chain-ladder forecasts are based on only 55 observations (from the upper triangle of a 10 by 10 matrix) compared to the micro-level analysis of the development of 5000 claims. Apparently, the chain-ladder methods uses exactly the correct set of summary statistics for the basis of its forecasts. From another viewpoint, this is precisely the result to be expected. The chain-ladder has been used successively by actuaries for decades and this collective wisdom is not to be ignored.

Does this result hold under "non-steady" environments? It is important to emphasize at this point that, for the most part, this study simulates reserving models that are applied mechanically and without subjective judgments. This is due to several reasons. First, actuaries' subjective judgment can vary considerably, largely depending on their professional experiences, which makes judgment hard to simulate. Second, the essential question we are trying to address is which model enables the data to tell the most. I.e., the scope of the study is to compare the reserving models' performance in predicting the outstanding liability by utilizing the same set of historical data. It is a fair-play in this sense. This does not imply that judgments are not important or not possible to be used in a micro-level model; rather, prior judgments could be easily incorporated with a Bayesian framework, and we believe that the proper use of judgments could improve the performance of both models, probably more so for the micro-level model since it is flexible to incorporate judgments at many different levels.

The simulated scenarios have demonstrated how predictions from different reserving methods are impacted by different environmental changes. The results suggest that, under some changing environments, there can be material errors in the chain-ladder reserve estimate whereas the micro-level model is able to generate reserve estimates with smaller reserve errors and higher precision using knowledge readily available to the insurer.

Particular attention should be drawn to the changing product mix scenario. As this type of change can only be well-measured by micro-level covariates, micro-level models would have the greatest advantage over macro-level models under this situation. The results for the Regulation Scenario and Changing Claims Processing Scenario share several features with those of Changing Product Mix Scenario, as all of these three scenarios simulate environmental changes that result in changes in the claim development speed.

In scenarios other than the changing product mix scenario or the mixed scenario, the environmental changes can be measured by incorporating time variables (AYs or CYs). In fact, macro-level models can also be extended to incorporate these time variables, see, e.g., Taylor (2014), but the large amount of individual claims data used by micro-level models makes it easier to estimate the impact of these factors.

The covariate used in the inflation scenario is only partially observable, so it provides some insight into the micro-level models' ability to deal with unobservable factors. It might be argued that the assumption of no prior information on the magnitude of inflation rates is unrealistic. As we emphasized earlier, the scope of this study is to compare the performance of various models with the same amount of historical data. The use of prior information on the claims inflation is typical, but for lines of business that are exposed to complex super-imposed inflation, the claims inflation patterns in the historical data can also have a great value for projecting the future inflation. In this scenario, the use of the micro-level model makes it easier to estimate the claims inflation in the past which may help the projection of inflation patterns in the future.

It might also be argued that the proposed micro-level model is guaranteed to provide better reserve estimates in this simulation study, as it is performed with good knowledge of the true underlying claims process. To get some insight into the impact of the model mis-specifications, we intentionally omit the covariates in the micro-level model. Even with such a big mis-specification, the micro-level model still outperforms the basic chain-ladder in most cases and even outperforms the trended chain-ladder in some cases. While we admit that the real-world results are not likely to be equally good due to the inevitable model risks, a point that we want to emphasize is that with such detailed individual claim level development data and extensive micro-level covariate information, actuaries should be able to fit a micro-level model that is at least close to the true underlying process.

4.2 Robustness Check

The Section 3 reported results are based on 5000 claims. To explore the impact of the number of claims on the results, we also experimented by using 50,000 and 500 claims for each sample in Case 3 of Scenario 2. For each model, although there are not any significant changes in the expected values of the reserve error, the standard deviations decrease proportionally with square root of the number of claims, i.e., $sd(RE) \propto 1/\sqrt{n}$, with n denoting the number of claims in each sample in our simulation study.

Another possible argument is that it is unfair to use the chain-ladder technique as a representative for macro-level models in the comparison since other methods (expected claims, B-F, etc.) are used by practitioners when they are aware of an unsteady environment. To address this potential argument, we applied the expected claims method to Case 3 of Changing Product Scenario. We assume the actuaries' expected total ultimate loss for the book of business, denoted by $\hat{E}(UL)$, is obtained by drawing 100 full-development samples (each with 5000 claims) from the population distribution and taking an average of the ultimate losses over the 100 samples. Under this setup, $\hat{E}(UL)$ could be regarded as a precise expectation of the ultimate loss. Outstanding liabilities are to be estimated for another 100 samples drawn from the population distribution. With $UL^{(a)}$ and $PL^{(a)}$ denoting the true ultimate loss and paid loss for the a^{th} sample respectively, the percentage reserve error for the a^{th} sample can be calculated through $RE^{(a)} = (UL^{(a)} - \hat{E}(UL))/(UL^{(a)} - PL^{(a)})$. The expected value and standard deviation of the percentage reserve error are 1.2% and 8.6% respectively. Although the expected claims method does not generate material errors in the reserve estimate, the reserve uncertainty is much larger compared to that of the micro-level model. The result suggests that the micro-level model would still be preferred even if the actuaries perform the expected claims technique with a precise expectation of the ultimate loss.

5 Concluding Remarks

This study compares forecasts generated by the basic chain-ladder method to those generated from a detailed micro-level model with parameters estimated using maximum likelihood estimation, a technique that is well-known for efficient use of data. Remarkably, the basic chain-ladder forecasts are comparable (only marginally poorer) to the micro-level forecasts in our simulation of a stable environment. When the changing environment causes different claims development patterns over accident years, the primary assumption of the chain-ladder technique no longer holds, resulting in material errors in the reserve estimates. The micro-level models, on the contrary, are able to efficiently identify and measure the impact of the environmental changes. The reserve estimates generated by the micro-level model do not appear to have material errors under any scenarios that we have studied. In addition, the use of extensive micro-level information reduces the reserve uncertainty, leading to reserve estimates with higher precision. The trending technique does help to reduce the material errors in the chain-ladder estimates, but it also introduces considerable additional variability to the reserve estimates.

The simulation results suggest that micro-level models are able to generate reserve estimates with better quality. This provides quantitative evidence to motivate the further investigation of the micro-level reserving. For actuaries responsible for setting reserves, this study highlights scenarios in which micro-level models outperform traditional macro-level models. Particular attention of the future research should be paid to loss reserving under a changing product mix for long-tail lines of business with a high degree of heterogeneity, for this type of environmental changes can only be efficiently handled by micro-level models.

The proposed micro-level model can be easily generalized to applications with empirical data. The hierarchical structure of the model provides great flexibility for modeling empirical claim development. Although we use certain distributional assumptions in this study, each block of the hierarchical model can be easily replaced with a different specification to conduct a sensitivity analysis with respect to the empirical data. By testing models with different specifications, a well-specified predictive model is likely to be obtained.

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6 Appendix 1: Detailed Simulation Procedure

Appendix 1 describes the detailed simulation procedure; Appendix 2 shows the parameters and covariates; and Appendix 3 documents the trending techniques for the trended chain-ladder method under each scenario.

6.1 The Population Distribution

The claims occurrence and development on the individual claim level is a complex process that contains many different events. The population distribution of the claims development process in our study is specified as a hierarchical structure with five blocks: (1) the claim occurrence time, (2) reporting delay, (3) transaction occurrence times, (4) transaction types, and (5) transaction-level payment amounts. An advantage of the hierarchical structure is that it would be flexible to use different specifications for each block. In the primary part of our simulation study, only reported claims are considered. We assume the claim occurrence time follows a uniform distribution; the reporting delay is zero; the transaction occurrence time follows a survival model characterized by time-dependent hazard rates; the transaction type follows a multinomial logit model; and the payment amount follows a log-normal distribution. Then the model is generalized to consider IBNR claims as well by assuming a Poisson distribution for the reporting delay. A hierarchical model is flexible to incorporate different covariates in each block, that is, some blocks could depend on covariates while others do not; and two blocks could depend on different sets of covariates. Particularly, we will incorporate covariates in the blocks for the reporting delay, transaction types and payment amounts, but not the other two blocks.

In order to specify the distributional assumptions for each block, we define the following notations for claim i.

- T_i = claim occurrence time (accident time),
- W_i = reporting time,
- S_i = settlement time,
- τ = valuation time (censoring time),
- D_{ij} = time of the jth transaction,
- E_{ij} = type of the jth transaction,
- P_{ij} = payment amount in the jth transaction,
- $U_i = W_i - T_i$ = reporting delay,
- $SD_i = S_i - W_i$ = settlement delay,
- τ_i = censoring time since notification,
- $V_{ij} = D_{ij} - W_i$ = time of the jth transaction since notification,
- $J_i = \sum_j \mathbb{1}\{V_{ij} \leq SD_i\}$ = number of transactions,
- $J_i^o = \sum_j \mathbb{1}\{V_{ij} \leq \tau_i\}$ = number of observed transactions as of the valuation time.

Under the five-block hierarchical specification, the likelihood for the full development process of claim i can be written as

$$L_i = f_{T_i} f_{U_i|T_i} f_{\boldsymbol{V}_i|T_i,U_i} f_{\boldsymbol{E}_i|T_i,U_i,\boldsymbol{V}_i} f_{\boldsymbol{P}_i|T_i,U_i,\boldsymbol{V}_i,\boldsymbol{E}_i}. \tag{1}$$

Claims Occurrence and Reporting. We use relatively simple assumptions for the claim occurrence time: claims can only occur at the end of each month, and the occurrence time follows a discrete uniform distribution. In most parts of the study, only reported claims are considered, so the reporting delay is assumed to be zero, i.e., a claim is reported right after the accident occurs. We will relax this assumption for one scenario in Section 3.7 by assuming a Poisson distributed reporting delay. With this modification, we are able to consider both the reported and IBNR claims. The modeling details for IBNR claims are documented in Section 5 of this Appendix.

Transaction Occurrence. For a given claim, the transaction occurrence times, also called payment lags in some literature, are determined by a survival model characterized by time-dependent hazard rates. We assume that the transactions can only occur at the end of a month, and there is no more than one transaction in each month. This discrete setup is consistent with the fact that many insurers aggregate transactions on a monthly basis by the end of each month. We distinguish the first transaction and the later transactions for each claim by using different hazard rates: $g(t)$ for the first transactions and $h(t)$ for the later ones. The time-dependent hazard rates $g(t)$ and $h(t)$ are specified as piece-wise constant functions on $(0, a_K]$ and $(0, b_L]$ respectively:

$$g(t) = \sum_{k=1}^{K} g_k \mathbb{1}\{a_{k-1} < t \leq a_k\},$$
$$h(t) = \sum_{l=1}^{L} h_l \mathbb{1}\{b_{l-1} < t \leq b_l\}.$$

Cumulative hazard rates $G(t)$ and $H(t)$ can be defined through

$$G(t) = \int_0^t g(s)ds,$$

$$H(t) = \int_0^t h(s)ds.$$

Particularly, we assume hazard rates jump every six months, i.e., $a_0 = 0$, $a_1 = 6$, ..., and $b_0 = 0$, $b_1 = 6$, ... Please note that the interval endpoints $\{a_K\}$ and $\{b_L\}$ are not distribution parameters. They will be explicitly specified in a later paragraph. The basic cumulative density functions of transaction occurrence times are given by

$$\begin{aligned}\Pr(V_1 \leqslant t) &= 1 - \exp\{-G(t)\},\\ \Pr(V_j \leqslant t) &= 1 - \exp\{-H(t)\}, \quad j > 1.\end{aligned}$$

The survival model used here is following Antonio and Plat (2012) except that it is now a discrete specification instead of a continuous one.

Additional restrictions are imposed in order to reduce the computational load. We assume that the first transaction can only occur in the first N_1 months since notification, and all claims are finalized within N_2 months since notification. That is, N_1 is regarded as the maximum waiting time to the first transaction, and N_2 is regarded as the maximum settlement delay. Hence, we can set $a_K = N_1$ and $b_L = N_2$. Under these additional assumptions, the probability that the first transaction occurs at time k, $k = 1, 2, ..., N_1$ is

$$\Pr(V_1 = k | V_1 \leq N_1) = \frac{\exp\{-G(k-1)\} - \exp\{-G(k)\}}{1 - \exp\{-G(N_1)\}}. \tag{2}$$

Given the occurrence time of the prior transaction, $V_{j-1} = s$, the probability that transaction j occurs at time k, $k = s+1, s+2, ..., N_2$ is

$$\Pr(V_j = k | V_{j-1} = s, V_j \leq N_2) = \frac{\exp\{-H(k-1)\} - \exp\{-H(k)\}}{\exp\{-H(s)\} - \exp\{-H(N_2)\}}, \quad j > 1. \tag{3}$$

In reality, the maximum waiting time to the first transaction and the maximum settlement delay highly depend on the lines of business. For short-tail lines such as auto material coverage, the first transaction usually occurs within several months and claims are usually settled within a year. For long-tail lines such as workers compensation, the first transaction could occur after a year since notification, and claims could develop for over a decade before they are settled. In the major part of this study, $N_1 = 36$ and $N = 120$ are adopted, which is intended to simulate a line of business with a relatively long-tail. With this assumption, we can now explicitly specify the intervals for the piecewise-constant hazard rates: $a_0 = 0$, $a_1 = 6$, ..., $a_6 = 36$, and $b_0 = 0$, $b_1 = 6$, ..., $b_{20} = 120$.

Transaction Types. We define three types of transactions: a "Type 1" transactions refers to settlement of claim without a payment; a "Type 2" transaction refers to settlement with a payment; and a "Type 3" transaction is an intermediate payment. That means, a Type 2 or 3 transaction contains a positive payment while a Type 1 transaction does not; a Type 1 or 2 transaction indicates the settlement of the claim and a Type 3 transaction means the claim will continue to develop. Given a transaction at time t, the transaction type is determined by a multinomial logit model. The probabilities depend on the time of the transaction and the covariate X. Again, we distinguish the first and later transactions by using different parameters for the multinomial logit model. With $m = 1, 2, 3$, the probabilities are given by

$$\Pr(E_1 = m) = \frac{\exp(\alpha_{m10} + \alpha_{m11}V_1 + \alpha_{m12}X)}{\sum_{m=1}^{3} \exp(\alpha_{m10} + \alpha_{m11}V_1 + \alpha_{m12}X)}, \tag{4}$$

$$\Pr(E_j = m) = \frac{\exp(\alpha_{m20} + \alpha_{m21}V_j + \alpha_{m22}X)}{\sum_{m=1}^{3} \exp(\alpha_{m20} + \alpha_{m21}V_j + \alpha_{m22}X)}, \ j > 1. \tag{5}$$

Transaction Amounts. We assume the payment amount associated with a Type 2 or 3 transaction follows a log-normal distribution with a location parameter μ that depends on the covariate X and a constant scale parameter σ. Let P_j denote the payment amount of the jth transaction. If the transaction is Type 1, then $P_j = 0$. Otherwise, P_j follows

$$log(P_j) \sim N(\mu, \sigma), \ \ \mu = \beta_0 + \beta_1 X. \tag{6}$$

Parameter Summary. Let $\boldsymbol{\theta}$ denote the vector of parameters for the population distribution. Then the complete list of parameters is shown as follows:

$$\begin{gathered}
\boldsymbol{\theta} = (\boldsymbol{g}, \boldsymbol{h}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3, \boldsymbol{\beta}, \sigma), \\
\boldsymbol{g} = (g_1, g_2, ..., g_K), \\
\boldsymbol{h} = (h_1, h_2, ..., h_L), \\
\boldsymbol{\alpha}_m = (\alpha_{m10}, \alpha_{m11}, \alpha_{m12}, \alpha_{m20}, \alpha_{m21}, \alpha_{m22}), \ m = 1, 2, 3, \\
\boldsymbol{\beta} = (\beta_0, \beta_1).
\end{gathered}$$

6.2 Sampling the Full Development Process

Each sample contains 5000 claims, drawn from the population distribution. We assume that a dataset of ten accident years is available and there are on average 500 claims for each accident year. The 500 claims could also be interpreted as the number of claims per exposure unit, so the model can be generalized to applications for increasing or decreasing underwriting volume.

Let $i = 1, 2, ...5000$ index the claims to be sampled. For claim i, sampling it from the population distribution could be regarded as the generation of the full development process, denoted by

$$\{T_i, (V_{ij}, E_{ij}, P_{ij}) : j = 1, 2, ..., J_i\},$$

where T_i is accident occurrence time, V_{ij}, E_{ij}, and P_{ij} are the time, type and payment amount of the jth transaction. The process is ended once a Type 1 or 2 transaction is drawn. Please note that for scenarios with a product mix, the claim-level covariate X_i needs to be generated as well. Please note that the development process of a claim is generated based on a timeline that starts from the claim reporting time. Each claim has its own development timeline since the reporting times are different. Converting to a common timeline that starts from the beginning of the first accident year, transaction j of claim i occurs at $D_{ij} = T_i + V_{ij}$.

After the generation of the full development process for each claim, losses are aggregated by accident year and development year to form a traditional run-off triangle (actually a run-off rectangle since the full development is known), following the standard actuarial practice. Macro-level models will be performed based on this aggregate data.

For claim i, the sampling procedure can be summarized as follows.

- The claim occurrence time T_i is drawn from a discrete uniform distribution with support $1, 2, ..., 120$.
- Generate the first transaction. Draw the occurrence time of the first transaction from (2). Draw the transaction type from (4). If the transaction type is 2 or 3, then draw the payment amount P_{i1} from (6), otherwise set $P_{i1} = 0$.
- Generate the later transactions. For $j = 2, 3, ...J_i$, draw transaction occurrence time V_{ij} from (3) and transaction type from (5). If the transaction type is 2 or 3, then draw payment amount P_{ij} from (6), otherwise set $P_{ij} = 0$.
- For any transaction, if the transaction type is 1 or 2, then the claim is settled by this transaction and the generation routine stops.

6.3 Estimation of Parameters in the Population Distribution

We consider a valuation date at the end of AY 10. The claims development drawn from the population distribution is censored by the valuation date. Let τ denote the censoring time on the timeline starting from the beginning of AY 1. Then $\tau = 120$. When converted to each claim's individual timeline that starts from the claim reporting time, the censoring time varies over different claims, denoted by τ_i for claim i. The censored development for a sample of 5000 claims is denoted by

$$\mathscr{H}_U = \{T_i, (V_{ij}, E_{ij}, P_{ij}) : i = 1, 2, ..., 5000,\ j = 1, 2, ..., J_i^o\}.$$

For the micro-level model, we estimate the distribution parameter $\boldsymbol{\theta}$ based on the past development $\mathscr{H}_U$. According to (1), the likelihood function for the observed development of claim i can be written as

$$L_i^o = f_{T_i} \times f_{U_i|T_i} \times f_{V_{i1}} \times (\prod_{j=2}^{J_i^o} f_{V_{ij}|V_{i,j-1}}) \times (\prod_{j=1}^{J_i^o} f_{E_{ij}|V_{ij}}) \times (\prod_{j=1}^{J_i^o} f_{P_{ij}|V_{ij},E_{ij}}) \tag{7}$$

While (1) shows the general format of the likelihood for a five-block hierarchical model, (7) is the more specific likelihood under our specifications of the model. As we only consider reported claims, the densities for the claim occurrence time and reporting delay, f_{T_i} and $f_{U_i|T_i}$ do not contain any parameters of interest. The other blocks of the likelihood are specified as follows: the likelihood of the first transaction is

$$f_{V_{i1}} = g(V_{i1}) \cdot \exp\{-G(\tau_i)\}; \tag{8}$$

the likelihood of the later transactions is

$$\prod_{j=2}^{J_i^o} f_{V_{ij}|V_{i,j-1}} = \prod_{j=2}^{J_i^o} h(V_{ij}) \cdot \exp\{-H(\tau_i)\}; \tag{9}$$

given the transaction times, the likelihood of the transaction types is

$$\prod_{j=1}^{J_i^o} f_{E_{ij}|V_{ij}} = \prod_{j=1}^{J_i^o} p_1^{\delta_{ij1}}(V_{ij}) \cdot p_2^{\delta_{ij2}}(V_{ij}) \cdot p_3^{\delta_{ij3}}(V_{ij}); \tag{10}$$

given the transaction types, the likelihood of the payment amounts is

$$\prod_{j=1}^{J_i^o} f_{P_{ij}|V_{ij},E_{ij}} = \prod_{j=1}^{J_i^o} f_P^{1-\delta_{ij1}}(P_{ij}). \tag{11}$$

Here $p_1(V_{ij})$, $p_2(V_{ij})$ and $p_3(V_{ij})$ are the probabilities that the jth transaction is Type 1, 2, and 3 respectively, following a multinomial logit specification; δ_{ijm} is an indicator variable that is 1 if the jth transaction is Type m, for $m = 1, 2, 3$; and f_P is the density function of the payment amounts, following a log-normal distribution.

Hazard rates can be estimated by maximizing the likelihood blocks (8) and (9). With a piece-wise constant specification of the hazard rates $g(t)$ and $h(t)$, the integrals in (8) and (9) can be expressed in a straightforward way and the maximizing of the likelihood can be done analytically. Optimizing over g_k, $k = 1, 2, ..., K$ or h_l, $l = 1, 2, ..., L$ leads to, respectively,

$$\hat{g_k} = \frac{N_{1k}}{\frac{\partial}{\partial g_k}\left(\int_{a_{k-1}}^{a_k} g(u)du\right)},$$

$$\hat{h_l} = \frac{N_{2l}}{\frac{\partial}{\partial h_l}\left(\int_{b_{l-1}}^{b_l} h(u)du\right)}.$$

Here N_{1k} is the number of the first transactions that occur in time interval $(a_{k-1}, a_k]$, and N_{2l} is the number of later transactions that occur in time interval $(b_{l-1}, b_l]$, i.e.,

$$N_{1k} = \sum_{i=1}^{5000} \mathbb{1}\{V_{i1} \in (a_{k-1}, a_k]\},$$

$$N_{2l} = \sum_{i=1}^{5000} \sum_{j=2}^{J_i^o} \mathbb{1}\{V_{ij} \in (b_{l-1}, b_l]\}.$$

The parameters related to transaction types, $\boldsymbol{\alpha}_m$ with $m = 1, 2, 3$, can be estimated by optimizing the likelihood block (10). The distribution for transaction types is specified as a multinomial logit model with a systematic component that depends on time and the covariate X_i, as shown in (4)-(5). The parameters can be estimated by the package “mlogit” in R.

The parameters related to payment amounts, i.e., $\boldsymbol{\beta}$ and σ, can be estimated by optimizing the likelihood block (11). The distribution for payment amounts is specified as a log-normal model with a location parameter that depends on the covariate X_i, as shown in equation (6). It is straightforward to use the package "glm" in R to estimate the log-normal parameters.

After the estimation routine, we get the MLEs of the distribution parameters

$$\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{g}}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{\alpha}}_1, \hat{\boldsymbol{\alpha}}_2, \hat{\boldsymbol{\alpha}}_3, \hat{\boldsymbol{\beta}}, \hat{\sigma}).$$

6.4 Monte-Carlo Valuation for the Future Development

With respect to the valuation time, the future development of the sampled claims is projected with Monte-Carlo valuation. In the Monte-Carlo procedure, pseudo-samples of the future development are drawn from the population distribution, but with estimated parameter $\hat{\boldsymbol{\theta}}$. For claim i that is not settled as of the valuation time τ_i, the occurrence time of the next transaction is determined by

$$\Pr(\hat{V}_{J_i^o+1} = k | \tau_i < \hat{V}_{J_i^o+1} \leq N_2) = \frac{\exp\{-\hat{H}(k-1)\} - \exp\{-\hat{H}(k)\}}{\exp\{-\hat{H}(\tau_i)\} - \exp\{-\hat{H}(N_2)\}}, \tag{12}$$

with $k = \tau_i + 1, \tau_i + 2, ..., N_2$. The occurrence times of the following transactions are determined by

$$\Pr(\hat{V}_{J_i^o+j} = k | \hat{V}_{J_i^o+j-1} = s, \hat{V}_{J_i^o+j} \leq N_2) = \frac{\exp\{-\hat{H}(k-1)\} - \exp\{-\hat{H}(k)\}}{\exp\{-\hat{H}(s)\} - \exp\{-\hat{H}(N_2)\}}, \tag{13}$$

with $j = 2, 3, ...$; $k = s + 1, s + 2, ..., N_2$; and $s > \tau_i$. The transaction type and payment amount for each transaction after the censoring time are determined by (5) and (6) respectively, except that the population parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \sigma)$ are now replaced with the estimated parameters $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, \hat{\sigma})$.

Given a sample of past development $\mathscr{H}_U^{(a)}$, we draw B pseudo-samples of the future development. The b^{th} pseudo-sample can be denoted by $\hat{\mathscr{H}}_L^{(a,b)} = \{(\hat{V}_{ij}, \hat{E}_{ij}, \hat{P}_{ij})^{(a,b)} : \hat{V}_{ij} > \tau_i, i = 1, 2, ..., 5000\}$. By adding up the payment amounts after the censoring date, we get an estimate of the outstanding liability, denoted by $\hat{R}_b^{(a)}$.

6.5 Modeling with IBNR Claims

In section 3.7, we extend the model to include IBNR claims for the changing product mix scenario by relaxing the assumption of zero reporting delay. Particularly, reporting delay is assumed to follow a Poisson distribution with parameter ρ:

$$U \sim \text{Poisson}(\rho), \quad \rho = \gamma_0 + \gamma_1 X,$$

where $\gamma_0 = 1$, and X is the same claim-level binary covariate that we used in section 3.2 ($X = 1$ for Type 1 claims and $X = 0$ for Type 2 claims). At first, we set $\gamma_1 = 0$ so that the covariate does not have an impact on the reporting delay, i.e., $\rho = 1$ for all claims. Then we set $\gamma_1 = 1$ to let the reporting delay depend on the covariate, i.e., $\rho = 2$ for Type 1 claims and $\rho = 1$ for Type 2 claims. When claim i is drawn from the population distribution, besides the sampling procedure described in section 2 of this Appendix, an additional step is needed to draw the reporting delay U_i from the Poisson distribution. The sampled full development process for claim i can be denoted by $\{T_i, U_i, (V_{ij}, E_{ij}, P_{ij}) : j = 1, 2, ..., J_i\}$.

With respect to a valuation date τ, claims in a sample can be categorized into three groups: incurred-but-not-reported (IBNR) claims, reported-but-not-settled (RBNS) claims, and settled claims.

- IBNR claim: $T_i + U_i > \tau$ and $T_i < \tau$. The development process is totally unobserved at time τ;
- RBNS claim: $T_i + U_i \leq \tau$ and $T_i + U_i + SD_i > \tau$. But SD_i is unobserved at τ, and the claim development process is censored at τ, i.e., only the patrial development $\{T_i, U_i, (V_{ij}, E_{ij}, P_{ij}) : j = 1, 2, ..., J_i^o\}$ is observed;
- Settled claim: $T_i + U_i + SD_i \leq \tau$. The full development process is observed.

RBNS and settled claims are often referred to as reported claims. For a reported claim, the density for the reporting delay, $f_{U_i|T_i} = e^{-\rho_i}\rho_i^{U_i}/(U_i!)$, needs to be added to the observed likelihood L_i^o in (7) for claim i. The total observed likelihood for the sample is obtained by multiplying Ł$_i^o$ over all the reported claims:

$$L^o = \prod_{i:\, T_i + U_i \leq \tau} L_i^o.$$

The MLEs of parameters, $\hat{\gamma}_0$ and $\hat{\gamma}_1$, can be easily obtained with a Poisson regression. Then the Poisson parameter ρ can be estimated by $\hat{\gamma}_0 + \hat{\gamma}_1$ for Type 1 claims and $\hat{\gamma}_0$ for Type 2 claims.

In the Monte-Carlo prediction routine for the future development, besides the procedure described in Section 4 of this appendix, an additional step is needed to generate the occurrence time and reporting delay for the IBNR claims. Let $n_{1,T}^{rep}$ and $n_{0,T}^{rep}$ denote the number of Type 1 and Type 2 reported claims that occur at time T, i.e., $n_{1,T}^{rep} = \sum_i \mathbb{1}\{T_i + U_i \leq \tau \;\&\; T_i = T \;\&\; X_i = 1\}$ and $n_{0,T}^{rep} = \sum_i \mathbb{1}\{T_i + U_i \leq \tau \;\&\; T_i = T \;\&\; X_i = 0\}$. With $F_U(.\;;\rho)$ denoting the cumulative density function of a Poisson distribution with parameter ρ, the number of Type 1 and Type 2 IBNR claims that occur at time T, $n_{1,T}^{ibnr}$ and $n_{0,T}^{ibnr}$, can be generated by

$$
\begin{aligned}
n_{1,T}^{ibnr} &= n_{1,T}^{rep} \times \frac{1 - F_U(\tau - T; \hat{\gamma}_0 + \hat{\gamma}_1)}{F_U(\tau - T; \hat{\gamma}_0 + \hat{\gamma}_1)}, \\
n_{0,T}^{ibnr} &= n_{0,T}^{rep} \times \frac{1 - F_U(\tau - T; \hat{\gamma}_0)}{F_U(\tau - T; \hat{\gamma}_0)},
\end{aligned}
$$

with $T = 1, 2, ..., \tau$. For an IBNR claim that occurs at T, the reporting delay is drawn from a truncated Poisson distribution with density $f_{U|U>\tau-T}$ and parameter $\hat{\rho}$, with $\hat{\rho} = \hat{\gamma}_0 + \hat{\gamma}_1$ for Type 1 claims and $\hat{\rho} = \hat{\gamma}_0$ for Type 2 claims.

Recall that we let τ_i denote the duration between the reporting time and the censoring time for claim i, i.e., $\tau_i = \tau - T_i - U_i$. Apparently, τ_i is negative for IBNR claims. For convenience, we set $\tau_i = 0$ for all IBNR claims. Then the transaction times of the IBNR claims can be predicted by (12) and (13), except that $\hat{H}(.)$ should be replaced with $\hat{G}(.)$ and N_2 should be replaced with N_1 in (12). The prediction of transaction types and payment amounts is the same as that used for the reported claims.

7 Appendix 2: Parameters and Covariates

Different scenarios can be simulated by adjusting the parameter $\boldsymbol{\theta}$ and using appropriate covariate X for the population distribution. Impacts on the claims development speed can be incorporated by adjusting $\boldsymbol{\alpha}$ in the multinomial logit model for transaction types (Equations (4) and (5)), and impacts on the payment amounts can be simulated by adjusting $\boldsymbol{\beta}$ in the log-normal model (Equation (6)). Among the these parameters, the

Scenario	Covariate
2. Changes in Product Mix	$X_i = \mathbb{1}\{\text{claim } i \text{ is from a construction company}\}$
4. Changes in Regulation	$X_i = \mathbb{1}\{AY_i \leq 5\}$
5. Changes in Claims Processing	$X_{ij} = \mathbb{1}\{CY_{ij} \leq 5\}$

Table 5: Covariates and Parameters of Interest for Scenarios 2, 4 and 5. The notation AY_i denotes the accident year for claims i, and CY_{ij} denotes the calendar year in which transaction j of claim i occurs.

Case	Covariate X	Covariate X^*	β_1	β_2
1. Stable Inflation	$X_{ij} = CY_{ij}$		$\neq 0$	0
2. Jump in Inflation	$X_{ij} = CY_{ij}$	$X^*_{ij} = \mathbb{1}\{CY_{ij} > 5\}$	$\neq 0$	$\neq 0$
3. Increasing Inflation	$X_{ij} = CY_{ij}$	$X^*_{ij} = CY^2_{ij}$	$\neq 0$	$\neq 0$

Table 6: Covariates and Parameters of Interest for Scenario 3

coefficients associated with the covariate X, α_{m12}, α_{m22}, and β_1, are the parameters of interest. All the parameters in $\boldsymbol{\theta}$ are explicitly specified for each scenario, but this appendix only provides details for the parameters of interest. All the other parameters in $\boldsymbol{\theta}$ are set to non-zero values.

Scenario 1: Steady Environment. No covariate is used in this scenario. To eliminate the covariates, we set α_{m12}, α_{m22} in (4)-(5) and β_1 in (6) to zero.

Scenarios 2, 4, 5: Changes in Product Mix, Regulations, and Claims Processing. In these scenarios, the environmental changes influence the claims development speed, but not the payment amounts. Table 5 documents the covariate of interest for each of these scenarios. The impact of the environmental changes is incorporated by setting α_{m21} and α_{m22} to non-zero values. The parameter β_1 is set to zero as the payment amounts are not influenced by the environmental changes.

Scenario 3: Inflation. In this scenario, an additional covariate X^* is incorporated in the log-normal distribution to generate the inflation structure. Instead of equation (6), the location parameter of the log-normal distribution is specified as

$$\mu = \beta_0 + \beta_1 X + \beta_2 X^*.$$

Different inflation structures are generated by defining different X^* and adjusting the parameters β_1, β_2. Table 6 shows the covariates and parameters of interest for each case in Scenario 3. With respect to a valuation date, the occurrence time of the future transactions are unknown, so the covariates are partially unobservable. As inflation does not influence the claims development speed, α_{m12} and α_{m22} are set to zero.

Scenario 6: Mixed Scenario. More than one covariate are used in this scenario: X_1 denotes the industry of the policy-holder, as defined in Table 5 for Scenario 2; X_2 denotes the claimant age; and X_3 indicates whether the injury type of the claim is exposed to medical inflation. Covariates X_1 and X_2 are used in the multinomial logit model for transaction types through

$$\begin{aligned}
\Pr(E_1 = m) &= \frac{\exp(\alpha_{m10} + \alpha_{m11}V_1 + \alpha_{m12}X_1 + \alpha_{m13}X_2)}{\sum_{m=1}^{3}\exp(\alpha_{m10} + \alpha_{m11}V_1 + \alpha_{m12}X_1 + \alpha_{m13}X_2)}, \\
\Pr(E_j = m) &= \frac{\exp(\alpha_{m20} + \alpha_{m21}V_j + \alpha_{m22}X_1 + \alpha_{m23}X_2)}{\sum_{m=1}^{3}\exp(\alpha_{m20} + \alpha_{m21}V_j + \alpha_{m22}X_1 + \alpha_{m23}X_2)}, \ j > 1.
\end{aligned}$$

Coefficients α_{m12}, α_{m13}, α_{m22}, and α_{m23} are set to non-zero values to incorporate the covariates' impact on the claims development speed. The other covariate X_3 is incorporated in the log-normal model for the payment amounts through

$$\mu = \beta_0 + \beta_1 CY + \beta_2 X_3 \cdot \mathbb{1}\{CY > 5\}.$$

The coefficients β_1 and β_2 are set to non-zero values to generate the inflation structure.

7.1 Appendix 3: Trending Techniques

For each sample of 5000 claims, the past history of the claims development is aggregated by accident year and development year to form a traditional run-off triangle:

$$\begin{bmatrix} L_{1,1} & L_{1,2} & \dots & L_{1,10} \\ L_{2,1} & L_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_{10,1} & 0 & \dots & 0 \end{bmatrix}, \tag{14}$$

where $L_{i,j}$ denotes the cumulative loss from AY i as of DY j and 0 denotes the unobservable cells as of the valuation date. A matrix of development factors (DFs) can be computed:

$$\begin{bmatrix} DF_{1,1} & DF_{1,2} & \dots & DF_{1,9} \\ DF_{2,1} & DF_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ DF_{9,1} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 \end{bmatrix}, \tag{15}$$

where $DF_{i,j} = L_{i,j+1}/L_{i,j}$.

An essential step of the chain-ladder technique is to project the DFs in the lower triangle based on the development patterns observed in the upper triangle. Under a changing environment, the observed development patterns represent a combination of the pure claims development and the impact from the environmental changes (the so-called "trend"). "Trending" techniques are often used to handle the impacts imposed by the environmental changes. As trending is an ad hoc activity that highly depends on actuaries' judgments, it would be impossible to test all the potential trending algorithms in my study. We assume that actuaries are aware of the type of "trend" that they are dealing with, but the magnitude of the trend needs to be estimated. We use a simple algorithm to estimate the trend magnitude with a rate, and then use the estimated trend rate to make appropriate adjustments to the historical data in the upper triangle. We understand that prior information about the trend magnitude is often used in reality, but to fit the scope of my simulation study (comparing the performance of models with the same amount of historical data), we apply the trending techniques only based on the historical data observed in the run-off triangles.

Trending techniques are used for Scenarios 2, 3 (Case 2 and 3), 4, and 5. We document the detailed trending algorithm for each scenario in this appendix. The documented trending algorithm stops when the DFs in the lower triangle are obtained, for the remaining steps to calculate the reserve estimates are common for each scenario. To perform a trending algorithm, the matrices of losses or DFs often need to be divided into several regions. Figure 8 shows the division of the matrix for each scenario, which will help to demonstrate how the trending algorithm is applied.

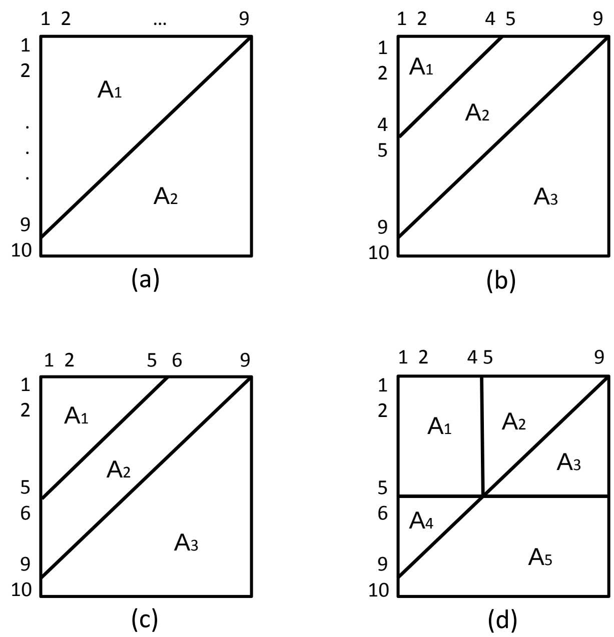

Figure 8: Division of Matrices. Panel (a): Scenario 2 (changes in product mix); Panel (b): Scenario 3 (inflation); Panel (c): Scenario 5 (changes in claims processing); and Panel (d): Scenario 4 (changes in regulation). Columns in a matrix are labeled by numbers on the top, rows in a matrix are labeled by numbers on the left.

Scenario 2: Changes in Product Mix.

Divide matrix (15) into two regions: $A_1 = \{(i, j) : i + j \leq 10\}$, and $A_2 = \{(i, j) : i + j \geq 11\}$. The regions are sketched by Panel (a) of Figure 8. On the aggregate level, the changing product mix imposes a gradual change on DFs over accident years. We measure this change by the ratio of the DFs for two adjacent accident years on A_1:

$$r_{i,j} = \frac{DF_{i,j}}{DF_{i-1,j}}, \quad (i, j) \in A_1.$$

The selected trend rate r, is the average of all the observable $r_{i,j}$'s on A_1:

$$r = \sum_{(i,j)\in A_1} r_{i,j}/n_{A_1},$$

where n_{A_1} is the number of r_{ij}'s on A_1. Starting from the latest observable DFs (latest diagonal) in matrix (15), the future DFs on A_2 can be calculated through

$$DF_{i,j} = r \cdot DF_{i-1,j}, \quad (i,j) \in A_2.$$

Scenario 3: Inflation.

The cumulative loss triangle (14) can be easily converted to an incremental loss triangle

$$\begin{bmatrix} C_{1,1} & C_{1,2} & \dots & C_{1,10} \\ C_{2,1} & C_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{10,1} & 0 & \dots & 0 \end{bmatrix}. \tag{16}$$

Under inflation, the incremental losses in (16) are expressed in nominal dollars. Let $\lambda(t)$ denote the annual inflation rate in calendar year t, if the inflation structure $\lambda(t)$ can be estimated, then the deflated incremental loss triangle can be obtained by converting each nominal loss $C_{i,j}$ to its value at the beginning of CY1, $C_{i,j}^{(r)}$:

$$C_{i,j}^{(r)} = \frac{C_{i,j}}{\{1+\lambda(1)\}\{1+\lambda(2)\}...\{1+\lambda(i+j-1)\}}.$$

Then a deflated cumulative loss triangle is easily constructed and the basic chain-ladder is applied to project the future development. At last, the deflated losses in the lower triangle are inflated back to their nominal values by using the inflation structure $\lambda(t)$.

Let λ denote the ratio of the incremental losses as of the same development age from two adjacent accident years, i.e., $\lambda_{i,j} = C_{i+1,j}/C_{i,j}$, then a matrix of λ's can be constructed:

$$\begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & \dots & \lambda_{1,9} \\ \lambda_{2,1} & \lambda_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{9,1} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 \end{bmatrix}. \tag{17}$$

Divide matrix (17) into three regions: $A_1 = \{(i,j) : i+j \le 5\}$, $A_2 = \{(i,j) : 6 \le i+j \le$

10}, and $A_3 = \{(i,j) : i+j \geq 11\}$, as shown in Panel (b) of Figure 8.

For Case 2, assume it is known that there is a jump in the inflation rate at the beginning of CY 6. We thus use λ's in CY 1-5 (those on A_1) to estimate the inflation rate before the jump, and λ's in CY 6-10 (those on A_2) to estimate the inflation rate after the jump. Then the time-varying inflation structure $\lambda(t)$ can be estimated by

$$\hat{\lambda}(t) = \begin{cases} \sum_{(i,j)\in A1} \lambda_{i,j}/n_{A_1}, & t \leq 5 \\ \sum_{(i,j)\in A2} \lambda_{i,j}/n_{A_2}, & t > 5, \end{cases}$$

where n_{A_1} is the number of $\lambda_{i,j}$'s on A_1, and n_{A_2} is the number of $\lambda_{i,j}$'s on A_2.

For case 3, assume it is known that the inflation structure $\lambda(t)$ is close to linear in t. A linear regression is conducted over all the observable $\lambda_{i,j}$'s in matrix (17), with calendar year $(i+j-1)$ being an independent variable:

$$\lambda_{i,j} = \delta_0 + \delta_1(i+j-1) + \epsilon_{i,j}, \quad (i,j) \in A_1 \cup A_2.$$

The inflation structure $\lambda(t)$ is then calculated with the regression coefficient estimates $\hat{\delta_0}$ and $\hat{\delta_1}$:

$$\hat{\lambda}(t) = \hat{\delta_0} + \hat{\delta_1} t.$$

Scenario 4: Changes in Regulation.

As shown in Panel (d) of Figure 8, divide matrix (15) into the following regions:

$$\begin{aligned} A_1 &= \{(i,j) : i \leq 5,\ j \leq 4\}, \\ A_2 &= \{(i,j) : i \leq 5,\ 5 \leq j \leq 10-i\}, \\ A_3 &= \{(i,j) : i \leq 5,\ 11-i \leq j \leq 9\}, \\ A_4 &= \{(i,j) : 6 \leq i \leq 9,\ j \leq 10-i\}, \\ A_5 &= \{(i,j) : 6 \leq i \leq 10,\ 11-i \leq j \leq 9\}. \end{aligned}$$

DFs are observable on A_1, A_2 and A_4, and need to be projected on A_3 and A_5. Assume it is known that DFs at the same development age are different for AY 1-5 and 6-10. With the chain-ladder technique, the projection of DFs on A_3 is based on the observable DFs on A_2, which is not affected by the regulation change. According to the basic chain-ladder algorithm, We first take the column averages of DFs on A_2 through

$$\overline{DF}_j = \sum_{i=1}^{10-j} DF_{i,j}/(10-j), \quad j = 5, 6, ..., 9,$$

and then set DFs on A_3 equal to the column averages:

$$DF_{i,j} = \overline{DF}_j, \quad (i,j) \in A_3.$$

Due to the regulation change, the observed historical development on A_1 and A_2 needs to be adjusted before being used for the projection on A_5. The column averages of DFs on A_1 and A_4 are calculated respectively through

$$\overline{DF}_j = \sum_{i=1}^{5} DF_{i,j}/5, \quad j = 1, 2, 3, 4,$$

$$\overline{DF^*}_j = \sum_{i=6}^{10-j} DF_{i,j}/(5-j), \quad j = 1, 2, 3, 4.$$

The impact of the regulation change on DFs is measured by a ratio r:

$$r = \frac{1}{4} \sum_{j=1}^{4} \frac{\overline{DF^*}_j}{\overline{DF}_j}.$$

DFs on A_1 and A_2 are then adjusted to the level after the regulation change by

$$DF^*_{i,j} = r \cdot DF_{i,j}, \quad (i,j) \in A_1 \cup A_2.$$

As all the observable DFs are adjusted to the level after the regulation change now, the basic chain-ladder algorithm can be applied to $DF^*_{i,j}$ on A_1, A_2 and A_4 to project the future development on A_5: take the column averages of the adjusted DFs on A_1, A_2 and A_4, and set DFs on A_5 equal to the column averages.

Scenario 5: Changes in Claims Processing.

Under this scenario, the DFs before and after the adoption of the new claims processing scheme are known to be different. Divide matrix (15) into three regions: $A_1 = \{(i,j) :$

$i + j \leq 6\}$, $A_2 = \{(i, j) : 7 \leq i + j \leq 10\}$, and $A_3 = \{(i, j) : i + j \geq 11\}$, as shown in Panel (c) of Figure 8. We then project the future claims development on A_3 only based on the observable DFs after the adoption of the new scheme, i.e., DFs on A_2. The column averages of the DFs on A_2 are calculated by

$$\overline{DF}_j = \sum_{i=7-j}^{10-j} DF_{i,j}/4, \quad j = 1, 2, ..., 6,$$

$$\overline{DF}_j = \sum_{i=1}^{10-j} DF_{i,j}/(10 - j), \quad j = 7, 8, 9,$$

and the future DFs on A_3 are set equal to the column averages:

$$DF_{i,j} = \overline{DF}_j, \quad (i, j) \in A_3.$$